

MATH COREQUISITES

MERGING CONTENT WITH ACTIVITY-BASED COURSES

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ACC PROFILE

- Multi-campus, single college district with 11 campuses
- 7,000-square-mile service area
- Enroll 70,000+ students annually (credit/CE/AE)
- ~80% Part-Time, 20% Full-Time

WHY USE COREQUISITES?

- Only 20% of students placed in developmental math have successfully completed a gateway course after 3 years ([CCRC Study](#))
- Students often don't sign up for math again, regardless of whether or not they were successful in their first developmental math course
- Corequisites give developmental students the chance to exit remediation and complete their gateway math course in one semester

WHERE WE STARTED

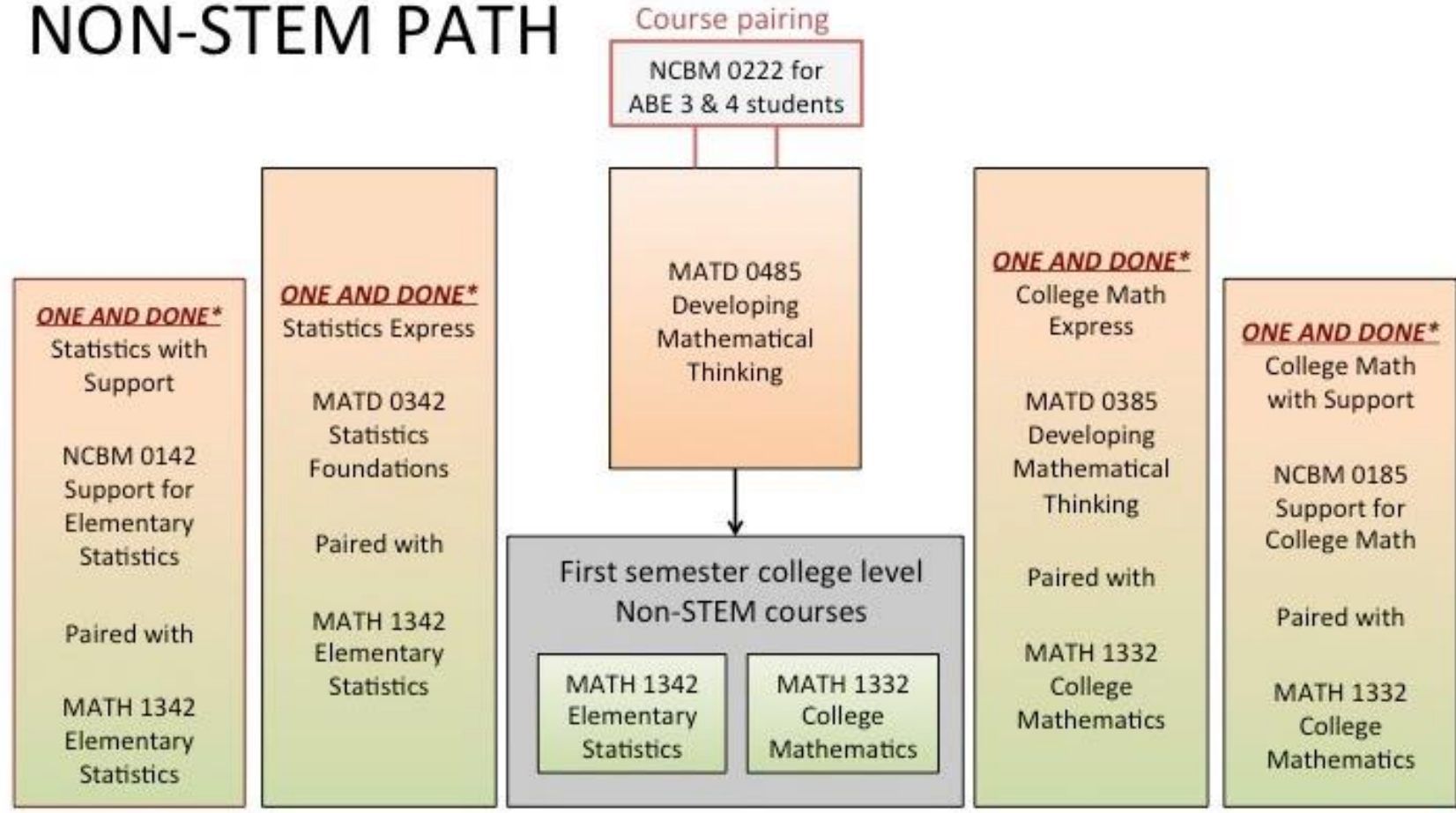
- Current data
 - Losing the most students in non-STEM
 - High non-STEM demand
- Current resources
 - Had developed accelerated non-STEM developmental course
 - Innovative faculty involved in non-STEM
- Potential for greatest impact

DEVELOPMENT STRATEGIES

- Requested access to data
- Administrative support
- Master plan for corequisite scaling for both STEM and non-STEM
- Looked to other schools with successful corequisites
- Assigned faculty leads
 - committee for each gateway course including full-time and adjunct faculty
- Determined two preparation levels and planned a corequisite for each group
- Worked with advising, financial aid and reporting to determine best structure
- Backward mapping – develop support needed for specific gateway course
- Faculty professional development – initial and ongoing

NON-STEM FLOWCHART

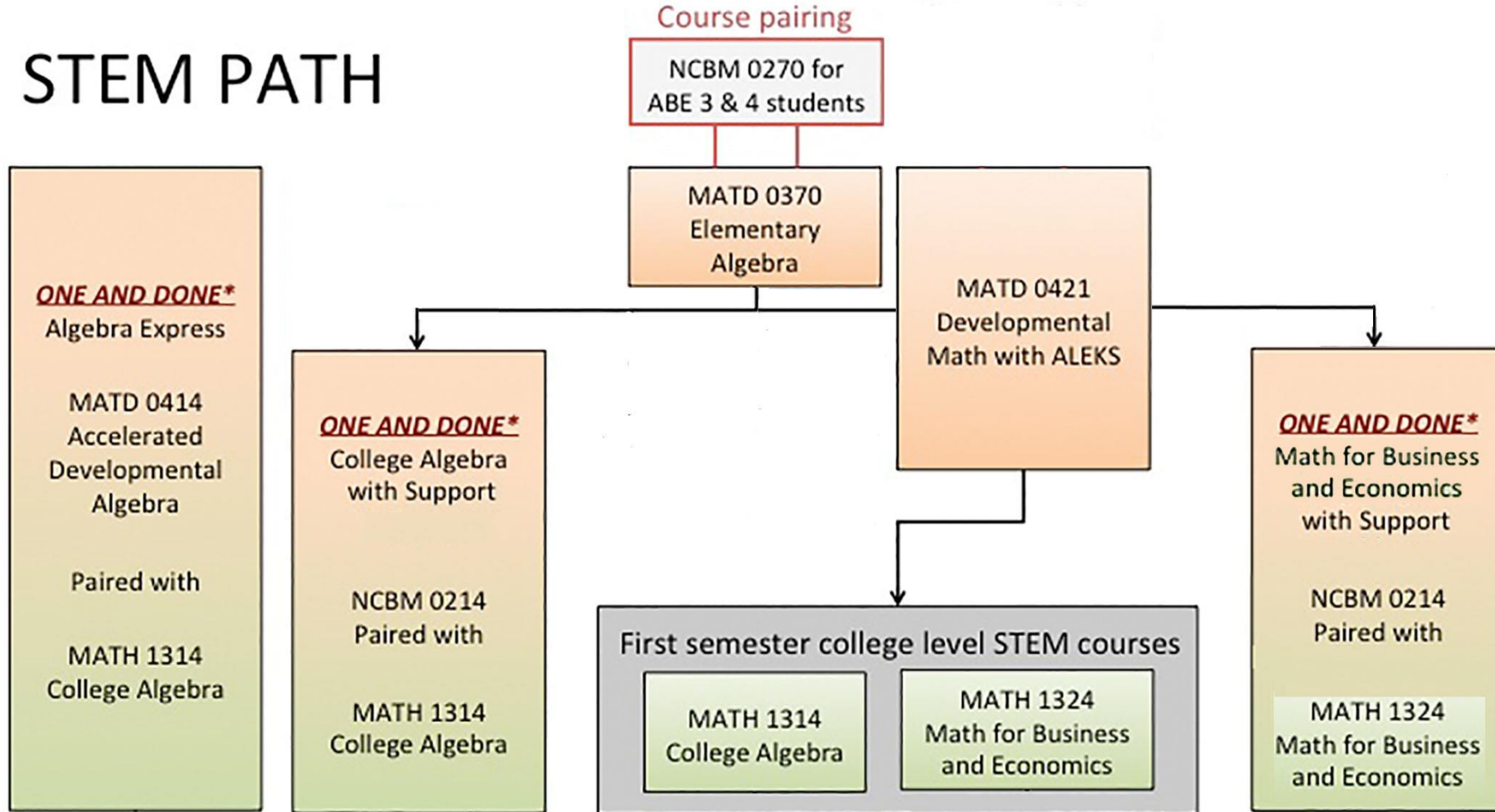
NON-STEM PATH



***ONE AND DONE:** These are co-requisite courses, combining developmental support with a 1st semester college course in one semester.

STEM FLOWCHART

STEM PATH



HIGHER PREPARATION LEVEL

- Mainstreaming model → gateway course has mix of developmental and college-level students
- Non-STEM – 4 credit hours
(1 hour support + 3 hour gateway course)
- STEM – 5 credit hours (2 support + 3 gateway)
- Support course
 - Meets before or after gateway course
 - Provides just-in-time support

LOWER PREPARATION LEVEL

- Developmental content fully integrated into gateway curriculum
- Non-STEM 6 credit hours (3 hour support + 3 hour gateway course)
- STEM 7 credit hours (4 support + 3 gateway)
- Two instructors co-teaching

LOWER PREP NON-STEM

- Collaborative Learning
- Active Learning
- Scaffolding

Contemporary Math Corequisite Activity

Section 2A: Unit Conversion

Students are introduced to the concept of units...



CME Unit Conversion Group Activity

The goal of the activity is to use dimensional analysis to convert units.

Introduction

- **Units** of a quantity describe what the quantity measures or counts.
 1. a) What units could you use if you were describing the distance from Austin to San Antonio? _____
b) What units could you use if you are buying a house and you want to know how large it is? _____
- We can describe units using words OR using an abbreviated form.

Example: When you are driving a car, your speed is read as miles per hour and written as mi/hr (or mph).
Words Abbreviated

 2. Based on the example, what math operation does the word “per” mean? _____
 3. Suppose you are buying some fabric. To calculate the unit price, you divide the price (in dollars) by the area (in square yards). The units are written as $\$/yd^2$.
Write the units using words: _____

Note: “square” corresponds to a 2 exponent on the units. What exponent will you use for “cubic”? _____

 4. The flow rate of a river is 5000 cubic feet per second. Write the units in abbreviated form: _____

Layering & Scaffolding...

Unit conversion (presented at the developmental level)

Unit Conversion

5. We know that 12 inches = 1 foot. This is an example of a **conversion factor** and can be written in three equivalent ways:

$$12 \text{ in} = 1 \text{ ft} \quad \text{or} \quad \frac{12 \text{ in}}{1 \text{ ft}} \quad \text{or} \quad \frac{1 \text{ ft}}{12 \text{ in}}$$

Notice that both fraction forms have a value in the numerator that is equal to the value in the denominator, so the fraction is equal to 1. These are called **unit fractions**.

- Write all three forms of the conversion factor we can use to convert between seconds and minutes.
6. Let's convert 20 feet per minute to feet per second. **Choose the correct conversion factor from #5.** Include units.

Since the minutes are in the denominator in the amount given, where should they be in the conversion factor so that the minutes cancel?

 Numerator/Denominator

$$\frac{20 \text{ feet}}{1 \text{ minute}} \cdot \frac{\quad}{\quad} =$$

Extra support early on
in the activity

Check your work:

- Did you start with the given value as a fraction?
- Did your units cancel?
- Did you end up with the correct units?
- Did you include the correct units in your final answer?

Your turn!



Practicing, deepening...

We are not given a conversion factor between inches and yards. Sometimes you will need to use more than one conversion factor in the problem. For example, we can convert inches to feet and then feet to yards.

Simplify the expression and find the answer:

$$\frac{180 \text{ in}}{1} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{1 \text{ yd}}{3 \text{ ft}} =$$

What do you notice about the units? _____

Are the units you are being asked to convert to in the numerator or denominator? _____

Additional support inserted as needed...

From the table, you get two different conversion factors between US dollars (\$) and British pounds (GBP). In the "Dollars per Foreign" column, the number 1.221 gives the conversion factor:

$$\$1.221 = 1 \text{ GBP} \quad \text{OR} \quad \frac{\$1.221}{1 \text{ GBP}} \quad \text{Think: } 1.221 \text{ dollars per } 1 \text{ GBP.}$$

What conversion factor comes from the number 0.8191 in the "Foreign per Dollar" column?

$$\text{Think: } 0.8191 \frac{\text{dollars or GBP}}{\text{dollars or GBP}} \text{ per } 1 \frac{\text{dollars or GBP}}{\text{dollars or GBP}} \quad \text{Conversion factor: } \frac{\text{dollars or GBP}}{\text{dollars or GBP}} = \frac{\text{dollars or GBP}}{\text{dollars or GBP}}$$

Suppose you are travelling from the United States to Europe.

a) Use the table to write two different conversion factors between the European euro and US dollars.

b) How many euros is \$200 worth?



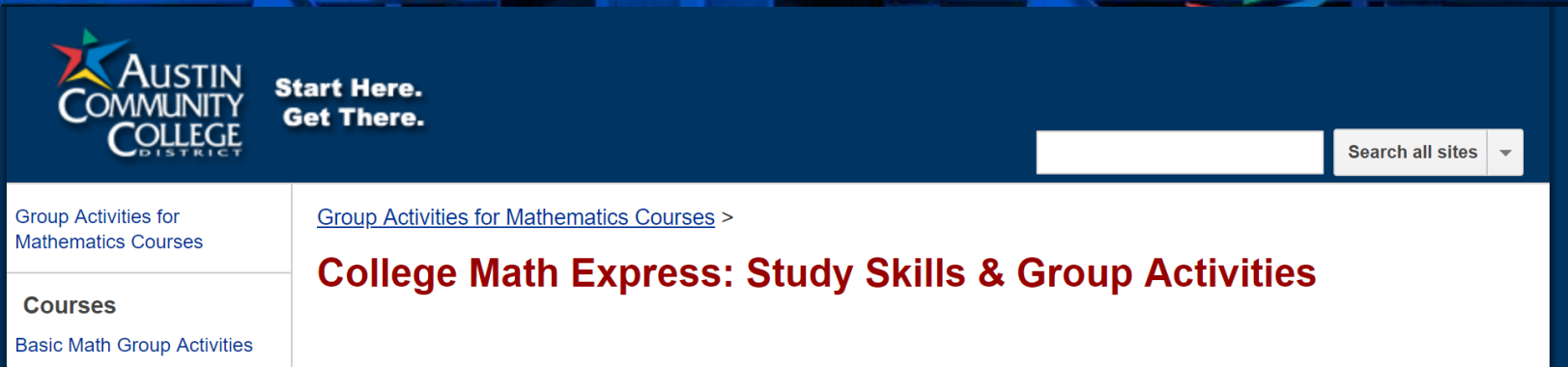
College-level Problem Solving!

Cantaloupes sell for 1.80 euros per kilogram in Belgium. What is the price in units of U.S. dollars per pound? Use the exchange rates in the table above and the conversion factor: 1 kg = 2.205 lb.

Organizing Instructor Resources

Group Activities Website

- Grouped by section
- Includes a brief description + link to subpage



Austin Community College District logo with the tagline "Start Here. Get There." and a search bar.

Group Activities for Mathematics Courses

Group Activities for Mathematics Courses >

College Math Express: Study Skills & Group Activities

Courses

Basic Math Group Activities

Section 2A	<p>CME Unit Conversion by Joey Offer and Marisa Bjorland</p> <p>This activity is similar to the MatD 0485 Unit Conversion activity, but with problems added from Bennett and Briggs. The activity uses a step-by-step guide to help students understand dimensional analysis. An introduction was added to cover unit abbreviations. Currency Conversion is at the end. (Metric units are included in this activity since they are included in a few of the 2A homework problems, but note that they aren't formally introduced until 2B).</p> <p>CME Square & Cubic Units, by Joey Offer and Marisa Bjorland</p> <p>This activity is similar to the MatD 0485 Square & Cubic Units activity, but with problems added from Bennett and Briggs. Students use units to help determine area and volume calculations. Two practice problems are included and a chart at the end summarizes what type of unit is used for the different types of measurements.</p>
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Activity Subpage

- Starts with textbook correlation
- Quick access to activity PDF file
- Time estimate based on pilot semester

CME Unit Conversion

Correlation: section 2A

- Unit conversions, conversion factors, Metric-UCSC conversions (2B), currency conversions
- does not include principles of unit analysis, conversions with units raised to powers, or the Understand-Solve-Explain process

Prerequisites: cancelling with fractions

Materials needed: copies of the activity [Download \(PDF\)](#).

Approximate time for the activity: 60-80 minutes

*Standardized Temperature Units is not on the Math 1332 recommended HW

Overview

The goal of the activity is to use unit analysis to convert units and solve problems.

This activity is similar to the MatD 0485 Unit Conversion activity, but with problems added from Bennett and Briggs. The activity uses a step-by-step guide to help students understand dimensional analysis. An introduction was added to cover unit abbreviations. Currency Conversion is at the end.

Activity Subpage

- Overview
- Before the Activity
- During the Activity
(Instructor Key included here)
- After the Activity

Before the activity.

Review canceling with fractions.

Examples:

$$\text{a) } 10m \cdot \frac{2n}{15m}$$

$$\text{b) } 8xy \cdot \frac{7}{16x}$$

Review converting a fraction to a decimal. Example: $28/9 = 3.111$

During the activity.

Answer Key: [Download](#)

Walk around and check that students are using the method of cancelling units to solve the problems. **Students may try to avoid using this method, especially on the simple one-step conversions.**

After the activity.

Go over the last page of problems as a class.

Have faster groups re-visit the original problem on the board and try to solve it using the techniques they have just learned. Go over it as a class.

Do the square and cubic units activity next.

Alternative example group activities website



- Grouped by section with specific learning outcomes identified
- Nice if there is a mix of shorter and longer activities to see coverage at a glance

<p><u>95% Rule, z-Scores, and Percentiles</u> Students work through use of the 95% Rule, then build the z-score as a concept before doing any calculations. Concludes with percentiles. Does not include the five number summary.</p>	Colleen Hosking	LO 2.3.3 LO 2.3.4-p
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Section 2.4 One Quantitative and One Categorical Variable

- LO 2.4.1 Identify outliers in a dataset based on the IQR method
- LO 2.4.2 Use a boxplot to describe data for a single quantitative variable
- LO 2.4.3 Use a side-by-side graph to visualize a relationship between quantitative and categorical variables
- LO 2.4.4 Examine a relationship between quantitative and categorical variables using comparative summary statistics

Activity (name, authors, description)	Authors	LOs covered
<p><u>Boxplots</u> Students practice reading and drawing boxplots as a visual representation of the numerical summaries for skewed data. The activity includes calculation of outlier fences and walks students through this step-by-step, including how to adjust the boxplot to flag potential outliers that fall beyond the fences. Concludes with use of technology and a look at unusual boxplots.</p>	Colleen Hosking Norma James Allison Sutton	LO 2.4.1 LO 2.4.2 LO 2.4.3 LO 2.4.4

Study Skills Support Materials

The following links to the activities we used to use in our MATD 0332 Basic Math with Study Skills course. There are some great options for use in our NCBM 0142 sections. Feel free to adapt them as needed.

Topics include:

- Student Commitment Contract
- Taking Good Notes
- Managing Time
- Preparing for Exams (Before, During, After)
- Managing Stress/Mindset

Activities site
also contains
study skills
support
materials



First Day of Class/Beginning of Semester	Grow your Brain Activity: Grow Your Brain Activity The goal of this activity is to have students read an article about how to grow your brain and help them better understand how they might learn math better.
	Vocabulary and Flash Cards: Vocabulary and flash cards by Mary Parker, with an alternate glossary version by Christy Dittmar and Marisa Bjorland.
Before Each Test	"Preparing for the Test" Activity: You can customize this reflective checklist to help your students prepare for each test. Download (Word) *This activity was modified from the 0332 materials for Kelly Holman's 1332 course. You will need to modify the activity to fit your course.
After Each Test	"Analyzing Test Mistakes" Activity: You can customize this activity to help students reflect on their test performance. This activity is not a test corrections activity. The activity assumes a detailed answer key has been posted for students to refer to. Download (PDF) . *This activity was modified from the 0332 materials for Kelly Holman's 1332 course. You will need to modify the activity to fit your course.
Mid-Semester	Mid-Semester Check-In Activity: Download (Word) A self-reflection activity designed to help students identify areas where they can improve habits, mindset, and behavior. Includes concrete goal-setting on second page. Modify page 1 to fit your instructional methods and unique problem areas with your group of students. Modify page 2 with grading scheme. (Only Word file is provided since modifications will be necessary.)
Before Test 1	A brief handout about Test Anxiety: Download (Word)

Statistics Corequisite Activity

Measures of Spread

Students begin the section with an activity on mean and standard deviation, then move into an activity on the 95% rule and z-scores.

Activity – Understanding the Standard Deviation, Section 2.3

Numerical Summaries

We have learned that graphs are a great way to see the “big picture” for a data set at a glance. They give us a general idea of the shape, center, and variation. Once we have this big picture, statisticians often like to look more closely at the center and variation of the data set. They do this by finding **numerical summaries**, which are calculations that give numbers we can use to represent the center and variation of a data set.

Measure of Center: Mean

Mean (or arithmetic average) = \bar{x}

This is a measure of center, interpreted as the “typical value” of a data set.

Step 1: Find the *sum* of all the values in your list.

Step 2: Divide by *n*, the number of values in your list.

$$\bar{x} = \frac{\text{sum}}{n}$$

Standard Deviation = *s*

This is a measure of variation, the “typical distance” of a data value to the mean of the data set. The standard deviation represents distance so it is always nonnegative.

***We will use technology to calculate the standard deviation, or it will be provided. ***

Your turn!



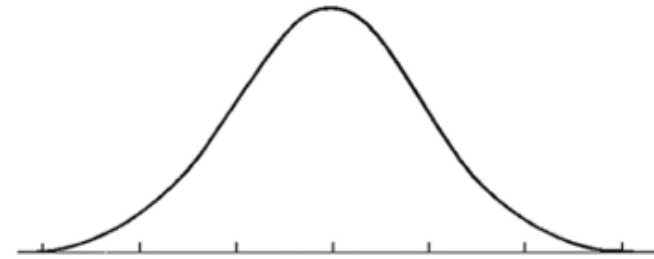
The 95% Rule, z-scores and Percentiles Activity

Getting an comfortable with standard deviation as a unit of measurement....

4. Suppose the heights of all men are approximately symmetric and bell-shaped with a mean of 70 inches and a standard deviation of 4 inches.

- a. Assume the tick marks are spaced a distance of 1 standard deviation apart. (Refer to Figure 2.18 on page 1.)

Label the mean height, and the heights that are 1, 2, and 3 standard deviations above and below the mean.



5. In statistics, we will use the variable "z" to represent the number of standard deviations a data value is from the mean. We will call this value a **z-score**.

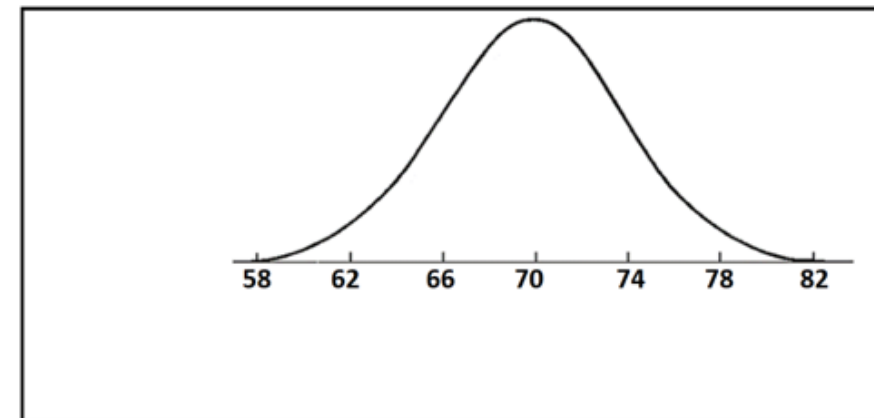
- a. The height 66 inches has a z-score of -1. Why?

- b. Find the z-scores for the following heights.

78 inches: $z =$ _____

58 inches: $z =$ _____

70 inches: $z =$ _____



Making the connection to z-scores....

Concluding with a statistics-level problem

A **z-score** is the number of standard deviations a data value is from the mean. We calculate z as follows:

$$z = \frac{\text{data value} - \text{mean}}{\text{standard deviation}}$$

For **samples**, this looks like: $Z = \frac{x - \bar{x}}{s}$

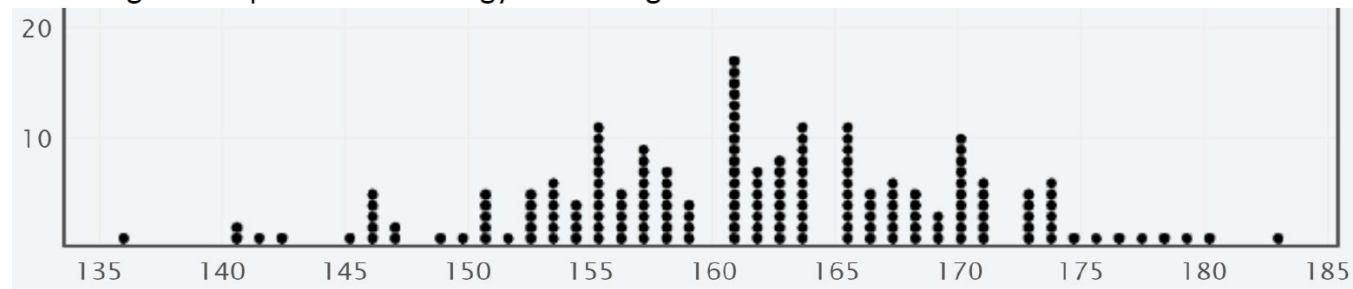
For **populations**, this looks like: $Z = \frac{x - \mu}{\sigma}$

Calculator Tip

Hit $\boxed{=}$ after subtracting.

Then divide.

6. The following is a dotplot of the same gymnast height data from Problem 1.



Suppose a gymnast from this group states their height is in the 90th percentile.

- What percent of the heights are *higher* than this gymnast's height? _____
- With 180 gymnasts, how many of the gymnasts are taller than this gymnast? _____
- Use the count in (b) and the dotplot to find the gymnast's height. _____
- Calculate the z-score for their height using the mean and standard deviation from Problem 1.
- Which is more unusual: A man with a height of 66 inches (Problem 5) or the height of this gymnast? How do you know?

Challenges & Successes

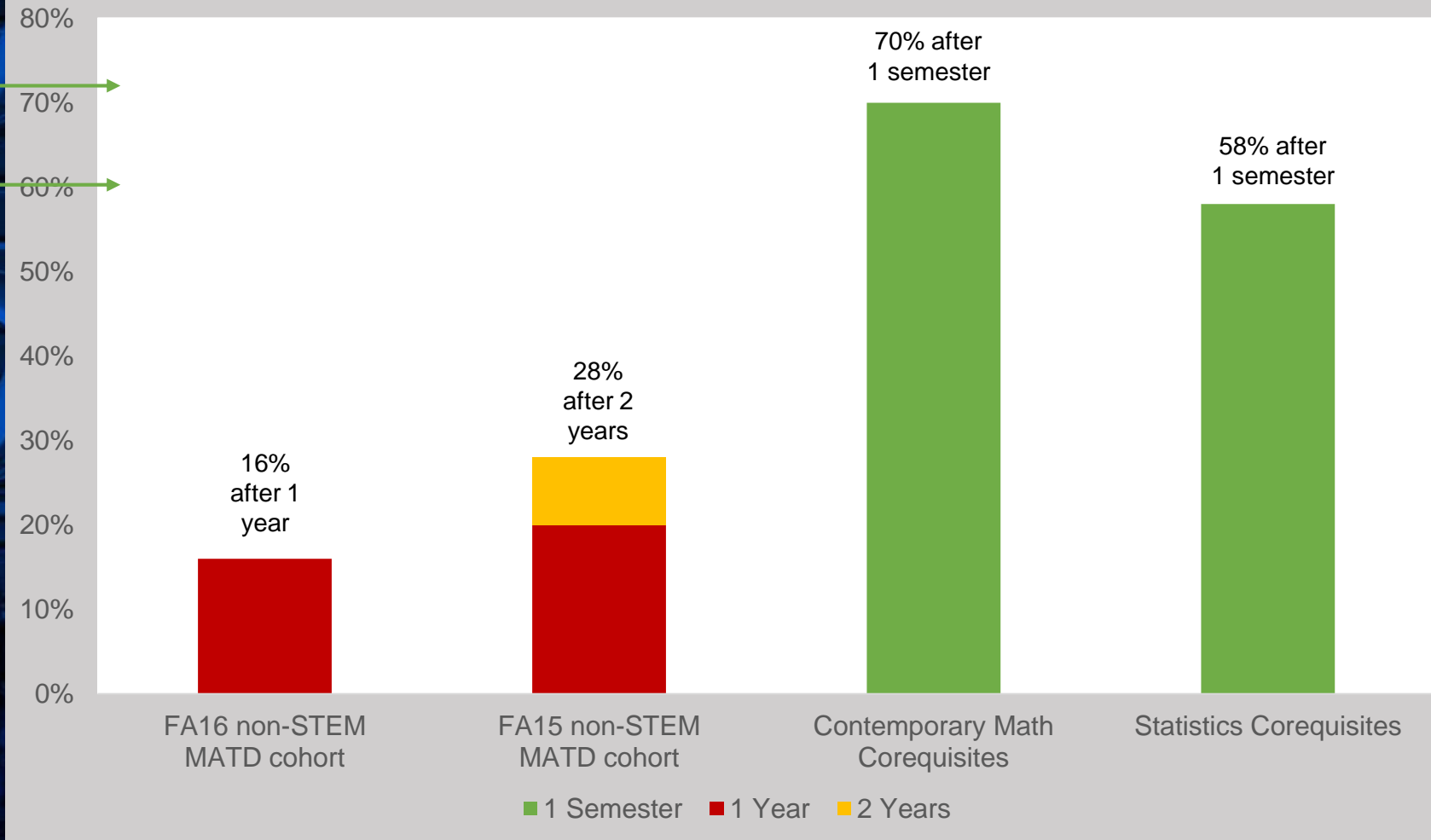
Success rates for college-level students in stand-alone sections

Cont. Math
71%

Statistics
60%



What percent of students completed a college-level math course?



Challenges



- Inexperience with using collaborative learning (Online Training)
- Insecurity with college-level content for some dev. math instructors
- Group pacing (monitoring group speed)
- Inadequate reading levels
- Writing complete sentences
- Team Teaching is hard!

Recommendations



- Fully integrate developmental math skills into college-level content
- Writing with your textbook in mind – it's their resource
- Use scaffolding on college-level content to help students achieve understanding through small steps
- Instructor website with activities & keys
- Ongoing mandatory trainings
- Faculty mentor/point-person
- Team Teaching training and planning recommendations
- Consider a reading/writing prerequisite
- Collaborate with advising, financial aid, and reporting areas

Thank you!

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Materials from this session available at:

<https://sites.google.com/a/austincc.edu/chosking/conference-materials>

CME Unit Conversion Group Activity

The goal of the activity is to use dimensional analysis to convert units.

Introduction

- **Units** of a quantity describe what the quantity measures or counts.
 1. a) What units could you use if you were describing the distance from Austin to San Antonio? _____
b) What units could you use if you are buying a house and you want to know how large it is? _____

- We can describe units using words OR using an abbreviated form.

Example: When you are driving a car, your speed is read as miles per hour and written as $\frac{\text{mi}}{\text{hr}}$ (or mph).
Words Abbreviated

2. Based on the example, what math operation does the word “per” mean? _____
3. Suppose you are buying some fabric. To calculate the unit price, you divide the price (in dollars) by the area (in square yards). The units are written as $\$/yd^2$.

Write the units using words: _____

Note: “square” corresponds to a 2 exponent on the units. What exponent will you use for “cubic”? _____

4. The flow rate of a river is 5000 cubic feet per second. Write the units in abbreviated form: _____

Unit Conversion

5. We know that 12 inches = 1 foot. This is an example of a **conversion factor** and can be written in three equivalent ways:

$$12 \text{ in} = 1 \text{ ft} \quad \text{or} \quad \frac{12 \text{ in}}{1 \text{ ft}} \quad \text{or} \quad \frac{1 \text{ ft}}{12 \text{ in}}$$

Notice that both fraction forms have a value in the numerator that is equal to the value in the denominator, so the fraction is equal to 1. These are called **unit fractions**.

- Write all three forms of the conversion factor we can use to convert between seconds and minutes.
6. Let’s convert 20 feet per minute to feet per second. **Choose the correct conversion factor from #5.** Include units.

Since the minutes are in the denominator in the amount given, where should they be in the conversion factor so that the minutes cancel?

_____ Numerator/Denominator

$$\frac{20 \text{ feet}}{1 \text{ minute}} \cdot \frac{\text{_____}}{\text{_____}} =$$

Check your work:

- Did you start with the given value as a fraction?
- Did your units cancel?
- Did you end up with the correct units?
- Did you include the correct units in your final answer?

Common Conversions:

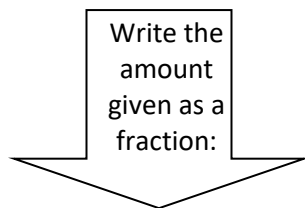
1 foot (ft) = 12 inches (in)	2 cups (c) = 1 pint (pt)
1 yard (yd) = 3 feet (ft)	2 pints (pt) = 1 quart (qt)
1 mile (mi) = 5280 feet (ft)	32 fluid ounces (fl oz) = 1 quart (qt)
1 year (yr) = 365 days	4 quarts (qt) = 1 gallon (gal)
1 day = 24 hours (hr)	16 ounces (oz) = 1 pound (lb)
1 hour (hr) = 60 minutes (min)	2000 pounds (lb) = 1 ton

UCSC-Metric Conversions:

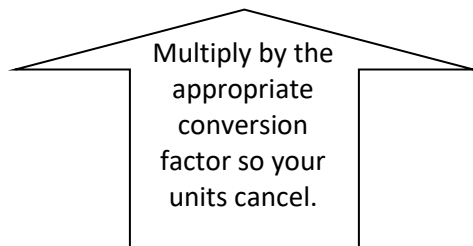
1 in = 2.540 cm
1 yd = 0.9144 m
1 mi = 1.6093 km
1 kg = 2.205 lb
1 qt = 0.9464 L
1 gal = 3.785 L

7. You plan to go to France, and you know the speedometer will be in kilometers per hour. You want to know what 60 miles per hour is in kilometers per hour.
- What speed are you asked to convert (the starting speed)? _____
 - What are the units you are asked to convert to? _____
 - Use the table to write down a conversion factor that might help convert from part a to part b. Write it in *three* forms.

 - Use the following steps to make the conversion. Round to one decimal place and include units.



_____ • _____ =



Check your work:

- Did you start with the given value as a fraction?
- Did your units cancel?
- Did you end up with the correct units?
- Did you include the correct units in your final answer?

8. Toddlers can drink a lot of milk! In one year, a toddler drinks about 12 gallons of milk. How many liters is this? Round to one decimal place. **Note:** Here the given value isn't a fraction (no "per"). We make it a fraction by writing it over 1.

$$\frac{12 \text{ gal}}{1} \cdot \underline{\hspace{2cm}} \approx$$

Check your work:

- Did you start with the given value as a fraction? Did you end up with the correct units?
 Did your units cancel? Did you include the correct units in your final answer?

9. **Practice.** Convert 16 liters to quarts. Round to one decimal place.

$$\underline{\hspace{2cm}} \cdot \underline{\hspace{2cm}} \approx$$

10. In celebration of National Cookie Day, the residents at *Sesame Street* baked a gigantic cookie for one of the characters on the show, Cookie Monster. The cookie was 180 inches in circumference. How many yards is the circumference of the cookie?
- What is the amount you are asked to convert? _____
 - What are the units you are asked to convert to? _____
 - We are not given a conversion factor between inches and yards. Sometimes you will need to use more than one conversion factor in the problem. For example, we can convert inches to feet and then feet to yards.

Simplify the expression and find the answer:

$$\frac{180 \text{ in}}{1} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{1 \text{ yd}}{3 \text{ ft}} =$$

What do you notice about the units? _____

Are the units you are being asked to convert to in the numerator or denominator? _____

To Convert Units:

Given Amount (If not a fraction, write over a 1.)	×	Conversion Factor #1* Did units cancel?	×	Conversion Factor #2* Did units cancel?	=	Converted Amount Are you left with the desired units?
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* Use more conversion factors as needed and write them so that units cancel and you end up with the desired units.

11. **Practice.** Convert 5 years to hours (neglecting leap years).

CME Unit Conversion Group Activity Answer key

The goal of the activity is to use dimensional analysis to convert units.

Introduction

- Units of a quantity describe what the quantity measures or counts.

- What units could you use if you were describing the distance from Austin to San Antonio? miles
 - What units could you use if you are buying a house and you want to know how large it is? square feet

- We can describe units using words OR using an abbreviated form.

Example: When you are driving a car, your speed is read as miles per hour and written as mi/hr (or mph).
Words Abbreviated

- Based on the example, what math operation does the word "per" mean? divide
- Suppose you are buying some fabric. To calculate the unit price, you divide the price (in dollars) by the area (in square yards). The units are written as $\$/yd^2$.

Write the units using words: dollars per square yard

Note: "square" corresponds to a 2 exponent on the units. What exponent will you use for "cubic"? 3

- The flow rate of a river is 5000 cubic feet per second. Write the units in abbreviated form: ft³/sec

Unit Conversion

- We know that 12 inches = 1 foot. This is an example of a conversion factor and can be written in three equivalent ways:

$$12 \text{ in} = 1 \text{ ft} \quad \text{or} \quad \frac{12 \text{ in}}{1 \text{ ft}} \quad \text{or} \quad \frac{1 \text{ ft}}{12 \text{ in}}$$

Notice that both fraction forms have a value in the numerator that is equal to the value in the denominator, so the fraction is equal to 1. These are called **unit fractions**.

- Write all three forms of the conversion factor we can use to convert between seconds and minutes.

$$1 \text{ min} = 60 \text{ sec.}$$

$$\frac{1 \text{ min.}}{60 \text{ sec.}}$$

$$\frac{60 \text{ sec.}}{1 \text{ min.}}$$

- Let's convert 20 feet per minute to feet per second. Choose the correct conversion factor from #5. Include units.

Since the minutes are in the denominator in the amount given, where should they be in the conversion factor so that the minutes cancel?

Numerator

Numerator/Denominator

$$\frac{20 \text{ feet}}{1 \text{ minute}} \cdot \frac{1 \text{ min.}}{60 \text{ sec.}} = \frac{20}{60} = \frac{2}{6} = \frac{1}{3} = \frac{0.3}{1} = \frac{0.3 \text{ ft.}}{1 \text{ sec.}}$$

Check your work:

- Did you start with the given value as a fraction?
- Did your units cancel?
- Did you end up with the correct units?
- Did you include the correct units in your final answer?

Common Conversions:

1 foot (ft) = 12 inches (in)	2 cups (c) = 1 pint (pt)
1 yard (yd) = 3 feet (ft)	2 pints (pt) = 1 quart (qt)
1 mile (mi) = 5280 feet (ft)	32 fluid ounces (fl oz) = 1 quart (qt)
1 year (yr) = 365 days	4 quarts (qt) = 1 gallon (gal)
1 day = 24 hours (hr)	16 ounces (oz) = 1 pound (lb)
1 hour (hr) = 60 minutes (min)	2000 pounds (lb) = 1 ton

UCSC-Metric Conversions:

1 in = 2.540 cm
1 yd = 0.9144 m
1 mi = 1.6093 km
1 kg = 2.205 lb
1 qt = 0.9464 L
1 gal = 3.785 L

7. You plan to go to France, and you know the speedometer will be in kilometers per hour. You want to know what 60 miles per hour is in kilometers per hour.

- What speed are you asked to convert (the starting speed)? 60 mph
- What are the units you are asked to convert to? Km per hour
- Use the table to write down a conversion factor that might help convert from part a to part b. Write it in *three* forms.

$$1 \text{ mile} = 1.6093 \text{ km}$$

$$\frac{1 \text{ mile}}{1.6093 \text{ km}}$$

$$\frac{1.6093 \text{ km}}{1 \text{ mile}}$$

d. Use the following steps to make the conversion. Round to one decimal place and include units.

Write the amount given as a fraction:

*Make sure students are filling in units in the conversion factors!

$$\frac{60 \text{ miles}}{1 \text{ hour}} \cdot \frac{1.6093 \text{ km}}{1 \text{ mile}} = \frac{96.558}{1} = 96.6 \text{ km/hr.}$$

Multiply by the appropriate conversion factor so your units cancel.

Check your work:

- Did you start with the given value as a fraction?
- Did your units cancel?
- Did you end up with the correct units?
- Did you include the correct units in your final answer?

8. Toddlers can drink a lot of milk! In one year, a toddler drinks about 12 gallons of milk. How many liters is this? Round to one decimal place. **Note:** Here the given value isn't a fraction (no "per"). We make it a fraction by writing it over 1.

$$1 \text{ gal.} = 3.785 \text{ L}$$

$$\frac{12 \text{ gal}}{1} \cdot \frac{3.785 \text{ L}}{1 \text{ gal.}} \approx 45.42 \text{ or } \boxed{45.4 \text{ L}}$$

Check your work:

Did you start with the given value as a fraction?

Did you end up with the correct units?

Did your units cancel?

Did you include the correct units in your final answer?

9. Practice. Convert 16 liters to quarts. Round to one decimal place.

$$1 \text{ qt.} = 0.9464 \text{ L}$$

$$\frac{16 \text{ L}}{1} \cdot \frac{1 \text{ qt.}}{0.9464 \text{ L}} \approx \frac{16}{0.9464} \approx 16.90617 \text{ or } \boxed{16.9 \text{ quarts}}$$

10. In celebration of National Cookie Day, the residents at *Sesame Street* baked a gigantic cookie for one of the characters on the show, Cookie Monster. The cookie was 180 inches in circumference. How many yards is the circumference of the cookie?

- a. What is the amount you are asked to convert?

$\boxed{180 \text{ inches}}$

- b. What are the units you are asked to convert to?

$\boxed{\text{yards}}$

- c. We are not given a conversion factor between inches and yards. Sometimes you will need to use more than one conversion factor in the problem. For example, we can convert inches to feet and then feet to yards.

Simplify the expression and find the answer:

$$\frac{180 \text{ in}}{1} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{1 \text{ yd}}{3 \text{ ft}} = \frac{180}{36} = \boxed{5 \text{ yd}}$$

What do you notice about the units?

$\boxed{\text{inches \& feet cancel}}$

Are the units you are being asked to convert to in the numerator or denominator?

$\boxed{\text{numerator}}$

To Convert Units:

Given Amount
(If not a fraction, write over a 1.)

×

Conversion Factor #1*
Did units cancel?

×

Conversion Factor #2*
Did units cancel?

=

Converted Amount
Are you left with the desired units?

* Use more conversion factors as needed and write them so that units cancel and you end up with the desired units.

11. Practice. Convert 5 years to hours (neglecting leap years).

$$\frac{5 \text{ years}}{1} \cdot \frac{365 \text{ days}}{1 \text{ year}} \cdot \frac{24 \text{ hours}}{1 \text{ day}} = \boxed{43,800 \text{ hours}}$$

Currency Conversions: Refer to the Currency Exchange Rate Table* for Questions 12 - 15:

Currency	Dollars per Foreign	Foreign per Dollar
British pound	1.221	0.8191
Canadian dollar	0.7586	1.318
European euro	1.058	0.9449
Japanese yen	0.008658	115.5
Mexican peso	0.04574	21.86

*Table 2.1 is from Bennett & Briggs "Using and Understanding Mathematics," 7th edition

12. From the table, you get two different conversion factors between US dollars (\$) and British pounds (GBP). In the "Dollars per Foreign" column, the number 1.221 gives the conversion factor:

$$\$1.221 = 1 \text{ GBP} \quad \text{OR} \quad \frac{\$1.221}{1 \text{ GBP}} \quad \text{Think: } 1.221 \text{ dollars per } 1 \text{ GBP.}$$

What conversion factor comes from the number 0.8191 in the "Foreign per Dollar" column?

Think: 0.8191 $\frac{1 \text{ GBP}}{\text{dollars or GBP}}$ per 1 $\frac{1 \text{ dollar}}{\text{dollars or GBP}}$ Conversion factor: $0.8191 \text{ GBP} = 1 \text{ dollar}$

13. Suppose you are travelling from the United States to Europe.
a) Use the table to write two different conversion factors between the European euro and US dollars.

$\$1.058 = 1 \text{ Euro}$ OR $\$1 = 0.9449 \text{ Euro}$ (OR written as unit fractions)

- b) How many euros is \$200 worth?

$$\frac{\$200}{1} \cdot \frac{1 \text{ Euro}}{\$1.058} =$$

14. Cantaloupes sell for 1.80 euros per kilogram in Belgium. What is the price in units of U.S. dollars per pound? Use the exchange rates in the table above and the conversion factor: 1 kg = 2.205 lb.

$$\frac{1.80 \text{ euros}}{1 \text{ kg}} \cdot \frac{1 \text{ kg}}{2.205 \text{ lb}} \cdot \frac{\$1.058}{1 \text{ euro}} = \$0.86 \text{ per pound} \text{ OR } 86 \text{ cents per pound}$$

15. A 0.8-liter bottle of Mexican wine costs 100 pesos. At that price, how much would a half-gallon jug of the same wine cost in dollars? Hint: First find the price of wine in units of U.S. dollars per gallon. ← "money" / volume

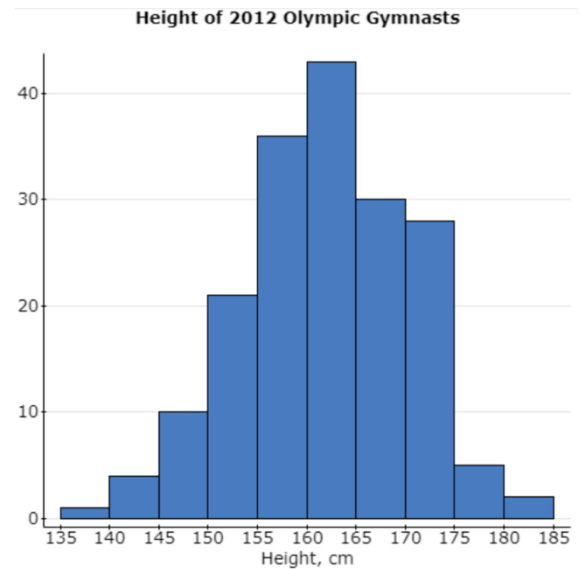
$$\frac{100 \text{ pesos}}{0.8 \text{ L}} \cdot \frac{3.785 \text{ L}}{1 \text{ gal.}} \cdot \frac{\$0.04574}{1 \text{ peso}} = \$21.64/\text{gal}$$

$$\frac{1}{2} (\$21.64) = \$10.82 \text{ for a } \frac{1}{2} \text{ gal.}$$

2.3 (Part 2): The 95% Rule, z-Scores, and Percentiles

1. The heights of gymnasts in the 2012 Olympics are shown in the histogram. The mean of these heights is 160.98 cm and the standard deviation is 8.52 cm.

- Mark the location of the mean on the horizontal axis. Draw vertical lines on your graph such that approximately 95% of the data falls between your lines. (This is just a quick eyeball estimate.)
- Calculate the height that is one standard deviation below the mean and the height that is one standard deviation above the mean.



$$160.98 - 8.52 = \underline{\hspace{2cm}} \text{ cm}$$

$$160.98 + 8.52 = \underline{\hspace{2cm}} \text{ cm}$$

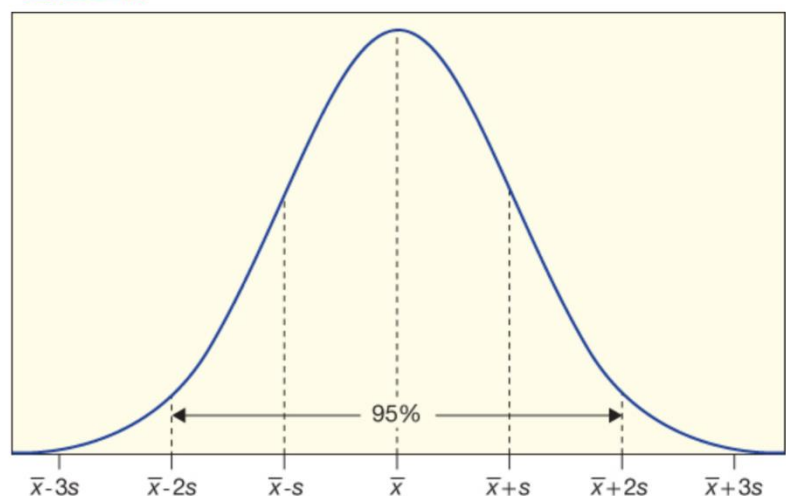
- Now calculate the heights that are **two** standard deviations from the mean. Mark these values on the horizontal axis.
- According to the raw data, 172 of the 180 gymnasts had heights within **two** standard deviations of the mean. Calculate the percent of the gymnasts who had heights in this interval.

The 95% Rule

If a distribution of data is approximately symmetric and bell-shaped, about _____% of the data should fall within two standard deviations of the mean.

How many standard deviations are between $\bar{x} - 2s$ and $\bar{x} + 2s$?

Figure 2.18 Most data are within two standard deviations of the mean

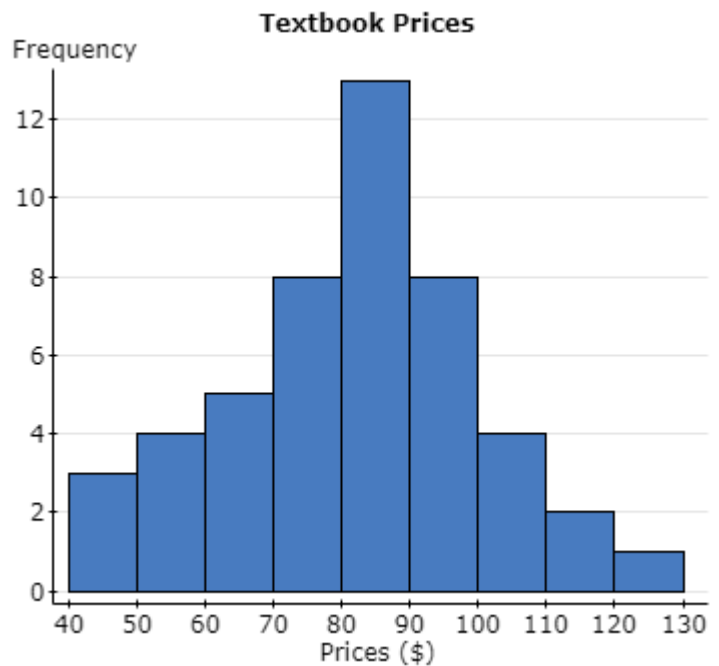


(Graphic Source: *Statistics: Unlocking the Power of Data*, by Lock, Lock, Lock, Lock, and Lock)

2. Consider this histogram of a sample of textbook prices.

- Estimate the mean: _____
Mark its location on the horizontal axis.
- Estimate an interval centered at your mean that contains approximately 95% of the data. Draw vertical lines on your histogram at these values.

_____ to _____

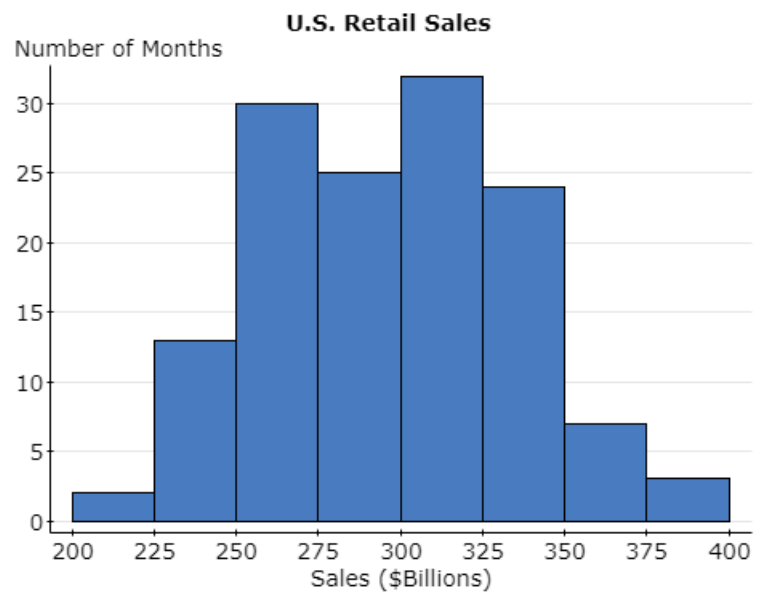


- Since this distribution is roughly bell-shaped and symmetric, about 95% of the textbook prices fall within _____ standard deviations of the mean. (See page 1.)

This tells us we can take the width of our interval in (b) and divide it by _____ to estimate the standard deviation. Use this method to estimate the standard deviation for textbook prices.

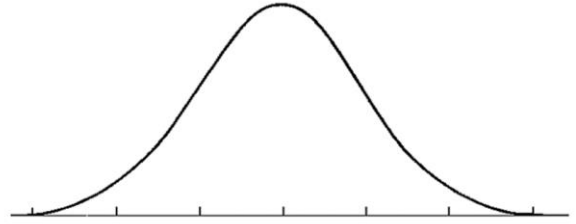
3. This distribution shows retail sales in the U.S. for the 136 months beginning January 2009.

- Estimate the mean and standard deviation.
- Have one person in your group open StatKey. Go to **One Quantitative Variable** and choose “Monthly Retail Sales” from the drop-down list of data sets. How do the actual mean and standard deviation compare with your estimate?



4. Suppose the heights of all men are approximately symmetric and bell-shaped with a mean of 70 inches and a standard deviation of 4 inches.

a. Assume the tick marks are spaced a distance of 1 standard deviation apart. (Refer to Figure 2.18 on page 1.) Label the mean height, and the heights that are 1, 2, and 3 standard deviations above and below the mean.



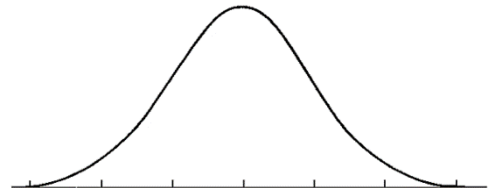
b. About 95% of men have heights between _____ and _____ inches. Shade the area under the curve that represents these men.



c. About what percent of men have heights *outside* the height interval in (b)? Shade the area under the curve that represents these men.



d. About what percent of men are shorter than 62 inches? Shade the area under the curve that represents these men.



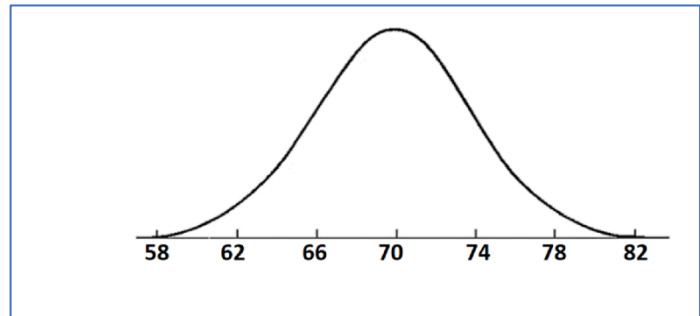
The **Pth percentile** is the value of the quantitative variable, like height, that is greater than P percent of the data.

e. Based on your answer for (d), men with heights of 62 inches therefore have a height in the _____ percentile.

f. A man has a height in the 60th percentile. Shade an estimated area under the curve that represents this scenario. Then estimate the man's height.



5. In statistics, we will use the variable “z” to represent the number of standard deviations a data value is from the mean. We will call this value a **z-score**. Recall from the previous page that the heights of all men are approximately symmetric and bell-shaped with a mean of 70 inches and a standard deviation of 4 inches



a. The height 66 inches has a z-score of -1. Why?

b. Find the z-scores for the following heights.

78 inches: $z =$ _____

58 inches: $z =$ _____

70 inches: $z =$ _____

c. Label the distribution above with the z-scores underneath their corresponding heights.

d. Use the graph to estimate the z-score for a height of 68 inches: $z \approx$ _____

A **z-score** is the number of standard deviations a data value is from the mean. We calculate z as follows:

$$z = \frac{\text{data value} - \text{mean}}{\text{standard deviation}}$$

For **samples**, this looks like: $Z = \frac{x - \bar{x}}{s}$

For **populations**, this looks like: $Z = \frac{x - \mu}{\sigma}$

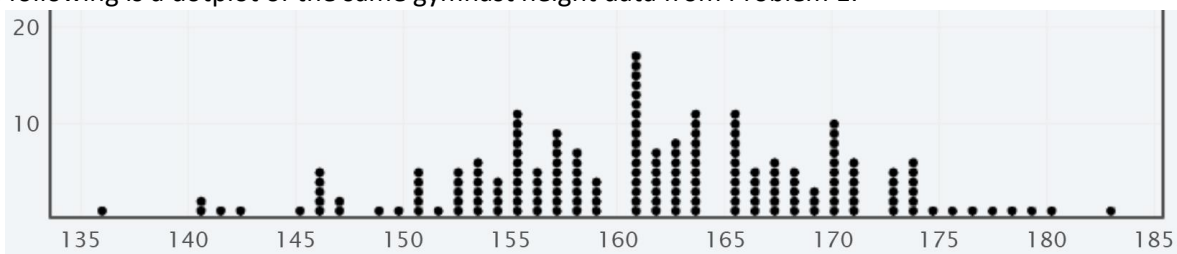
e. Use the formula to verify the z-score you estimated in (5d).

Calculator Tip

Hit [=] after subtracting.

Then divide.

6. The following is a dotplot of the same gymnast height data from Problem 1.



Suppose a gymnast from this group states their height is in the 90th percentile.

a. What percent of the heights are *higher* than this gymnast’s height? _____

b. With 180 gymnasts, how many of the gymnasts are taller than this gymnast? _____

c. Use the count in (b) and the dotplot to find the gymnast’s height. _____

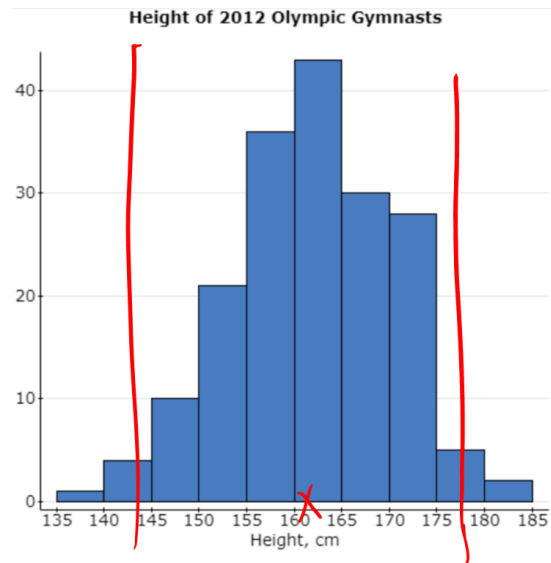
d. Calculate the z-score for their height using the mean and standard deviation from Problem 1.

e. Which is more unusual: A man with a height of 66 inches (Problem 5) or the height of this gymnast? How do you know?

2.3 (Part 2): The 95% Rule, z-Scores, and Percentiles

1. The heights of gymnasts in the 2012 Olympics are shown in the histogram. The mean of these heights is 160.98 cm and the standard deviation is 8.52 cm.

- Mark the location of the mean on the horizontal axis. Draw vertical lines on your graph such approximately 95% of the data falls between your lines. (This is just a quick eyeball estimate.)
- Calculate the height that is one standard deviation below the mean and the height that is one standard deviation above the mean.



$$160.98 - 8.52 = \underline{152.46} \text{ cm}$$

$$160.98 + 8.52 = \underline{169.5} \text{ cm}$$

c. Now calculate the heights that are **two** standard deviations from the mean. Mark these values on the horizontal axis.

$$160.98 - 2(8.52) = 143.94$$

$$160.98 + 2(8.52) = 178.02$$

d. According to the raw data, 172 of the 180 gymnasts had heights within **two** standard deviations of the mean. Calculate the percent of the gymnasts who had heights in this interval.

$$p = \frac{172}{180} \approx 0.956 = 95.6\%$$

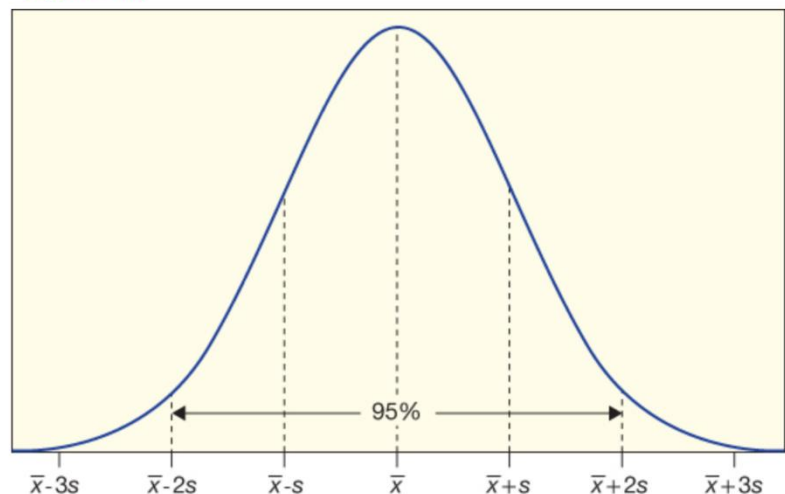
The 95% Rule

If a distribution of data is approximately symmetric and bell-shaped, about 95% of the data should fall within two standard deviations of the mean.

How many standard deviations are between $\bar{x} - 2s$ and $\bar{x} + 2s$?

4

Figure 2.18 Most data are within two standard deviations of the mean



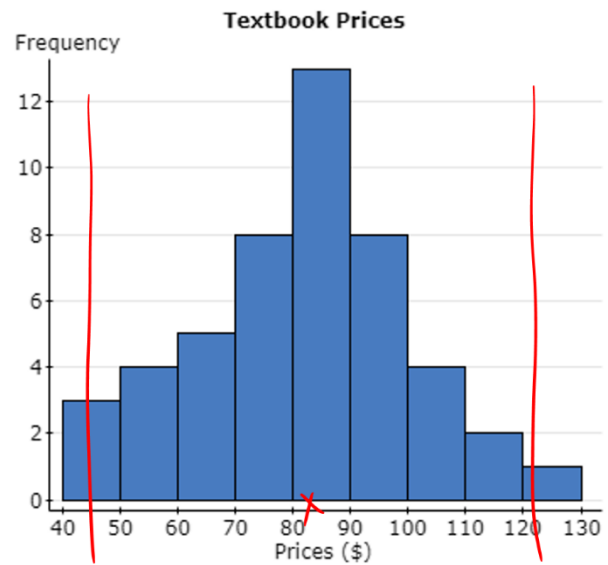
(Graphic Source: *Statistics: Unlocking the Power of Data*, by Lock, Lock, Lock, Lock, and Lock)

2. Consider this histogram of a sample of textbook prices.

(Estimates will vary.)

- Estimate the mean: \$82
Mark its location on the horizontal axis.
- Estimate an interval centered at your mean that contains approximately 95% of the data. Draw vertical lines on your histogram at these values.

\$42 to \$122



- Since this distribution is roughly bell-shaped and symmetric, about 95% of the textbook prices fall within 2 standard deviations of the mean. (See page 1.)

This tells us we can take the width of our interval in (b) and divide it by 4 to estimate the standard deviation. Use this method to estimate the standard deviation for textbook prices.

$$\$122 - \$42 = \$80 \rightarrow \$80/4 = \$20 \quad \text{OR} \quad \$82 - \$42 = \$40 \rightarrow \$40/2 = \$20$$

The standard deviation is estimated to be about \$20.

3. This distribution shows retail sales in the U.S. for the 136 months beginning January 2009.

- Estimate the mean and standard deviation.

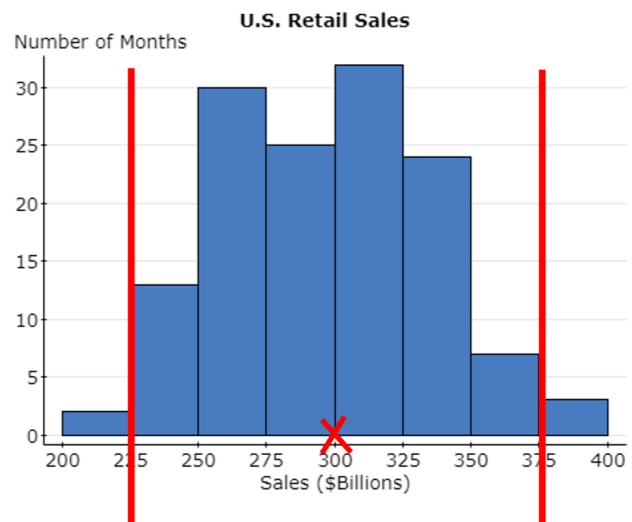
$$\bar{x} \approx \$300 \text{ billion}$$

$$\text{Four stdev} = \$375\text{B} - \$225\text{B} =$$

$$\$150\text{B}$$

$$\text{One stdev} = \$150\text{B}/4 = \$37.5\text{B}$$

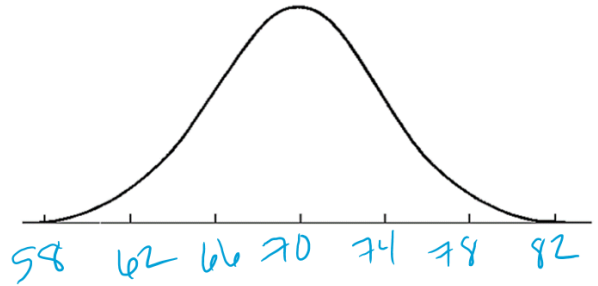
- Have one person in your group open StatKey. Go to **One Quantitative Variable** and choose "Monthly Retail Sales" from the drop-down list of data sets. How do the actual mean and standard deviation compare with your estimate?



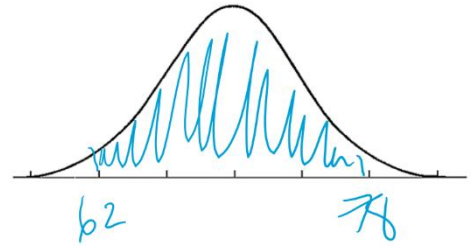
$\$37.971\text{B} \rightarrow$ Very close to the estimate!

4. Suppose the heights of all men are approximately symmetric and bell-shaped with a mean of 70 inches and a standard deviation of 4 inches.

- a. Assume the tick marks are spaced a distance of 1 standard deviation apart. (Refer to Figure 2.18 on page 1.) Label the mean height, and the heights that are 1, 2, and 3 standard deviations above and below the mean.

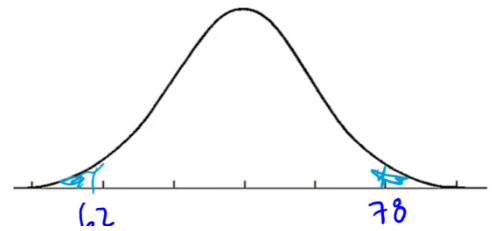


- b. About 95% of men have heights between 62 and 78 inches. Shade the area under the curve that represents these men.



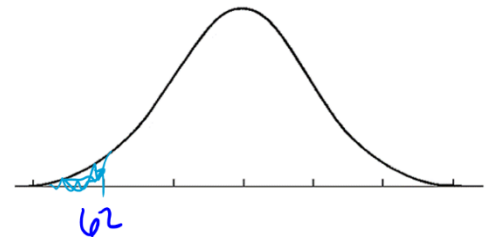
- c. About what percent of men have heights *outside* the height interval in (b)? Shade the area under the curve that represents these men.

$100\% - 95\% = 5\%$



- d. About what percent of men are shorter than 62 inches? Shade the area under the curve that represents these men.

$5\%/2 = 2.5\%$

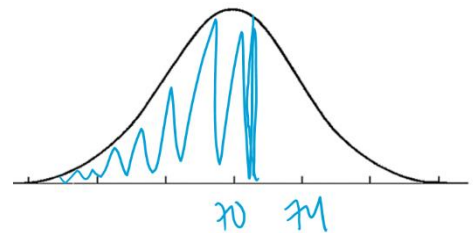


The **Pth percentile** is the value of the quantitative variable, like height, that is greater than P percent of the data.

- e. Based on your answer for (d), men with heights of 62 inches therefore have a height in the 2.5th percentile.

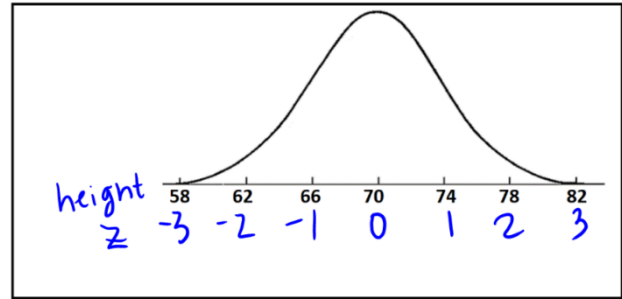
- f. A man has a height in the 60th percentile. Shade an estimated area under the curve that represents this scenario. Then estimate the man's height.

About 71 inches



5. In statistics, we will use the variable “z” to represent the number of standard deviations a data value is from the mean. We will call this value a **z-score**. Recall from the previous page that the heights of all men are approximately symmetric and bell-shaped with a mean of 70 inches and a standard deviation of 4 inches

- a. The height 66 inches has a z-score of -1. Why?
It is one standard deviation below the mean.



- b. Find the z-scores for the following heights.

78 inches: $z = \underline{\quad 2 \quad}$

58 inches: $z = \underline{\quad -3 \quad}$

70 inches: $z = \underline{\quad 0 \quad}$

- c. Label the distribution above with the z-scores underneath their corresponding heights.

- d. Use the graph to estimate the z-score for a height of 68 inches: $z \approx \underline{\quad -0.5 \quad}$

A **z-score** is the number of standard deviations a data value is from the mean. We calculate z as follows:

$$z = \frac{\text{data value} - \text{mean}}{\text{standard deviation}}$$

For **samples**, this looks like: $Z = \frac{x - \bar{x}}{s}$

For **populations**, this looks like: $Z = \frac{x - \mu}{\sigma}$

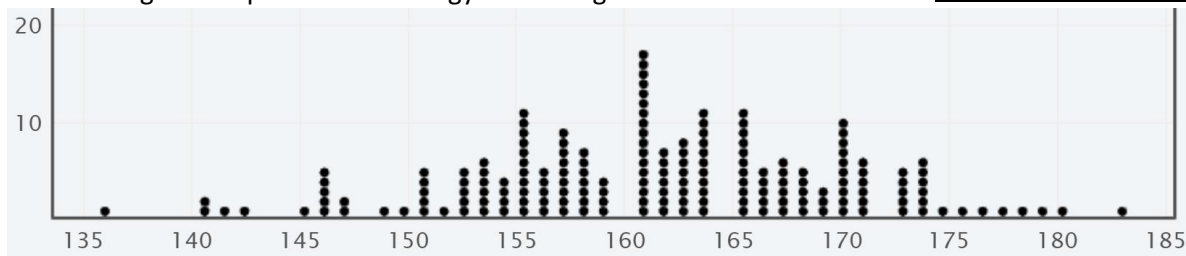
- e. Use the formula to verify the z-score you estimated in (5d).

$$z = \frac{68 - 70}{4} = \frac{-2}{4} = -0.5$$

Calculator Tip

Hit $\boxed{=}$ after subtracting.
 Then divide.

6. The following is a dotplot of the same gymnast height data from Problem 1.



Suppose a gymnast from this group states their height is in the 90th percentile.

- a. What percent of the heights are *higher* than this gymnast’s height? **$\underline{\quad 10\% \quad}$**
 b. With 180 gymnasts, how many of the gymnasts are taller than this gymnast? **$\underline{\quad 10\% \text{ of } 180 = 18 \quad}$**
 c. Use the count in (b) and the dotplot to find the gymnast’s height. **$\underline{\quad 173 \text{ cm} \quad}$**
 d. Calculate the z-score for their height using the mean and standard deviation from Problem 1.

$$z = \frac{173 - 160.98}{8.52} = \frac{12.02}{8.52} \approx 1.41$$

- e. Which is more unusual: A man with a height of 66 inches (Problem 5) or the height of this gymnast? How do you know? **The gymnast’s height is more unusual since it is farther from the mean (higher absolute z-score).**