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Articles should relate to issues that inform and broaden our understanding and practice of teaching and learning in developmental education. The subject of the article may emphasize innovative approaches, best practices, how meaningful research effects teaching and learning, or techniques to enhance student performance.

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The deadline for inclusion in in the fall 2015 issue is Sept. 15, 2015.

IN THIS ISSUE

Assessing Developmental Students' Number Sense: A Case Study

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Abstract

The data for this study were gathered from an assignment consisting of 10 number sense related mathematics problems completed in an algebra course at developmental level. The results of the study suggest that a majority of developmental mathematics students use routine algorithmic procedures rather than mathematical reasoning to solve problems. They lack quantitative judgment or estimation skills. Only a small percentage of students utilize benchmarks to mentally compare numbers and are unable to use numbers in convenient ways to simplify calculations. A socio-constructive approach to teaching that encourages multiple procedures for problem-solving and invites students to invent and discuss ways to solve a numerical situation is recommended in this study. Educators are encouraged to question, facilitate, and engage rather than model solutions to their students.

Mathematics proficiency is becoming increasingly important, especially for the technological careers of the 21st century. As a result, employers are demanding higher levels of mathematics skills from their employees in their workplaces than in the past. In addition, mathematics and science-oriented jobs will have the highest rate of growth and tend to yield higher salaries. Therefore, students with limited mathematics proficiency may have limited career opportunities (Lago & Diperna, 2010). However, many students perceive mathematics as a subject that is not creative and is disconnected from reality, consisting of facts and symbols that need to be memorized and have no value (Maclellan, 2012; Silver, 1989). These students do not perceive that meaningful ways of mathematics learning increase logical thinking and problem-solving skills. As increasing numbers of students are entering colleges, a vast majority of them are leaving high school underprepared for college-level work especially in mathematics. The rising number of college students enrolling in developmental mathematics courses suggests a need to reexamine the mathematics curriculum and the learning environment that developmental educators and secondary school teachers provide for these students. Most of these students are taught to follow routine algorithms to derive answers and are lacking quantitative thinking skills, including number sense, the skills of which provide the basic foundation for higher order mathematical skills and concepts (Lago & Diperna, 2010).

What is Number Sense?

Number sense is a holistic construct that is difficult to define (Yang & Wu, 2010).It is not a fixed entity that a student either has or does not have but rather a process that develops and matures with experience and knowledge (Reys & Yang, 1998; Sood & Jitendra, 2007).Some other definitions of number sense include "the ability to quickly understand, approximate, and manipulate numerical quantities" (Wilson et al, 2009, p.124) and "a term which encompasses several skills related to a 'common sense' about numbers" (Gay & Aichele, 1997, p. 27). Students with good number sense can invent their own procedures for conducting numerical operations.They have a good sense of numerical magnitude, can represent the same number in multiple ways and can recognize major numerical errors (Gersten et al., 2005).In other words, students with good number sense have the ability to use numbers in flexible ways to make mathematical judgment and to develop useful strategies for handling numbers and operations. "Those who view numbers in this way continually utilize a variety of internal 'checks and balances' to judge the reasonableness of numerical outcomes.When an outcome conflicts with the perceived expectation, the person revisits the mathematical situation to externally view it, often through another lens, attempting to resolve the conflict" (Reys et al., 1999, p. 61).

Students with strong number sense understand how to:

- use numbers in flexible ways when adding, subtracting, multiplying or dividing;
- use benchmarks to make mathematical judgments;
- make mental calculations and reasonable estimations;
- make predictions;
- understand numerical relationships between mathematical concepts, facts and skills;
- recognize unreasonable answers. (Faulkner, 2009; Dunphy, 2007; Gersten & Chard, 1999; Gersten et al., 2005; Lago & Diperna, 2010;; Malofeeva et al., 2004; Reys et al., 1999).

These students are capable of transferring their mathematical knowledge and skills to a broad range of quantitative tasks.They find mathematics everywhere, not only in school.

"Number Sense" continued

Purpose of the Study

An increasing number of studies have focused on fostering number sense among students. Most of these studies have examined improving instructional models for teachers, early intervention efforts, and ways of improving instruction.

The targeted populations in these studies included elementary students, students with learning disabilities and low income, and middle school students (Bobis, 2008; Cain, 2009; Dunphy, 2007; Faulkner, 2009; Lago & Diperna, 2010; Malofeeva et el., 2004; Sood & Jiterdra, 2007;Wilson et al., 2009). In addition, cross-cultural studies have been conducted including those in sites such as Taiwan, Japan, Australia, and Sweden (Reys & Yang, 1998; Yang & Wu, 2010; Reys et al.,1999). Because of the importance of number sense skills, it is essential that developmental mathematics educators investigate how to improve the learning environments of their struggling students in a way that increases number sense. Therefore, the primary purpose of this study was to analyze the number sense of students enrolled in a developmental mathematics course offered by the developmental studies department of a Pennsylvania university.

The secondary purpose of this study was to apply the results of the analysis to improve instructional practices of developmental mathematics educators in fostering the growth of number sense in their students.

Sample

Subjects in this study were students enrolled in a developmental mathematics course titled Elements of Algebra. Students were placed in this course based on their standardized Compass mathematics test scores, which they took during their freshman orientation. They scored above 30 in Pre-Algebra and between 0 and 26 in Algebra.

They were instructed to solve 10 number sense related problems in the mathematics lab during the spring semester of 2010. The author developed these problems after reviewing literature on number sense. The students solved these problems in the mathematics laboratory without using calculators. It should be noted here that students in these mathematics classes were required to attend at least 10 tutorial/ mathematics laboratory sessions during the semester, and their attendance in these sessions was included in the grade calculation. Participation in this study was voluntary. Students received bonus points and one mathematics lab attendance grade to encourage participation in this study. In addition, participating students provided demographic information which is summarized in the following table:

Table 1: Demographic Information

Gender/Race/Generation	Distribution	Sample	Percent
Gender	Male	14	48
	Female	15	52
	Total	29	100
Race	White	15	52
	African American	10	34
	Hispanic	1	4
	Asian	3	10
Generation in College	First	17	59
	Second	7	24
	More than second	4	14
	Did not answer	1	3

Findings

Student responses to the 10 number-sense-related basic arithmetic problems are described below. Students were also instructed to explain their responses in writing.

Q1. What is three tenths more than 0.52?

This question was designed to investigate whether the students understood the positional notation that characterizes our base-10 number system, their word names and basic mathematics vocabulary. Their responses are summarized in Table 2.

Table 2: Q1 Responses

Response type	Number	Percent
Correct answer	12	41
Incorrect answer	17	59

"Number Sense" continued

Out of 29 students, only 12 (41%) students answered the question correctly (.52 + .3 = .82). Eight of them did not offer any explanations. One student explained, "I would first convert 3/10 into a decimal and then I would add that decimal to the one given to get my sum." Some of the particular incorrect responses included the following: "you multiply 0.52 by three"; "you have to divide by 10"; "I personally don't even remember how to do this. I do remember how 2 is in ones column & 5 is in the tenth column." Another student wrote "0.52 + 0.03 = 0.55."

It appears from analysis of the above that most of these students had a weak understanding of the place value recognition of numbers in base 10. Some of these students most likely were not familiar with very basic mathematics vocabulary, such as "more."

Q2. Is $3\frac{1}{7} - 1\frac{5}{7}$ the same as $3\frac{3}{7} - 2$? Why or why not?

This problem was designed to investigate whether the students had a conceptual understanding of fractions and flexibility with numbers, especially fractions to simplify calculation. Table 3 provides a summary of their responses.

Table 3: Q2 Responses

Response type	Number	Percent
Yes	7	24
No	22	76

Seven out of 29 (24%) students concluded "yes" to this question. Out of the seven, one student followed a routine algorithm for subtraction of fractions and concluded that they are equal. Two students provided explanations: "I am not allowed to use a calculator. I think yes it's the same because $1\frac{5}{7}$ rounded to 2 is the same." Another student wrote, "Yes they are equal to each other. I do not know how to explain why, they are just are?" Out of the 22 students who responded "no," most incorrectly subtracted the fractions.

It appears from this study that most of the students struggled with both conceptual and procedural understanding of fractions. Moreover, they were most likely not used to using numbers in flexible ways to make their computations simpler. Research demonstrates that there is a correlation between mastering fractions and performance in advanced mathematics courses. Students who struggle with fractions most likely will not graduate from college and will have fewer career opportunities (Jordan et al., 2013; Seigler & Pyke, 2013).

Q3. Which is larger: $\frac{13}{24}$ or $\frac{19}{39}$?

This problem was intended to explore the students' conceptual understanding of rational numbers, especially whether they were able to compare the size of rational numbers by using benchmarks. Table 4 represents the summary of their responses.

Table 4: Q3 Responses

Response type	Number	Percent
13/24 is larger	23	79
19/39 is larger	5	17
Equal	1	3

Out of these, 23 (79%) were correct in stating that 13/24 is larger than 19/39; 14 of them did not provide any explanations. Seven students explained their answers:

"I would initially say that 13/24 is larger because the bottom of the fraction is smaller."

13/24 is more than 1/2 compared to 19/39, half of 24 is 12 so 13/24 bigger than 19/39."

Another similar response was, "13/24 is greater because it is more than one half."

The students used 1/2 as their benchmark. One student drew the two fractions and concluded from the diagram. Three more students indicated that 13/24 is larger than 19/39 without using a benchmark. "Because the two numbers are closer making cluster to a whole." "13/24 is larger because dividing it 24 time only it will be a bigger number than dividing 19 - 39 times." "13/24 is larger because 13 can go into 24 more than 19 into 39."

Some of the respondents who incorrectly indicated that 19/29 is larger than 13/24 offered the following explanations: "because 39 is more than 24" and "because 19 > 13 and 39 > 24." Surprisingly, one student indicated that the two fractions are equal. It appears from the analysis that a very small percentage of students in this study were able to use strategies that utilized one or more components of number sense (e.g., benchmarks, number magnitude) to mentally compare the two given fractions.

Q4. Find the best estimate of 1125 /0.98.

This problem was designed to identify students' understanding of the meaning and effect of operations (identify more or less than 1125). The students' responses to this problem are summarized in Table 5.

"Number Sense" continued

Table 5: Q4 Responses

Response type	Number	Percent
Reasonable estimation (number sense-based)	8	28
Did not answer	3	10
Wrong estimation	4	14
Did not understand Question	1	3
Paper pencil/calculator/explanation without answers	13	45

He then added 22.55 to 1125 and wrote the estimation as 1147.55. Three students used paper and pencil division. Out of these three students, only one student successfully came up with the correct quotient. Two could not divide them properly.

Very poor performance of students was observed with this item. A large proportion of the students were most likely lacking knowledge of the relative effects of mathematical operations on numbers, especially with decimal numbers. They were also deficient in reasonable estimation skills. They most likely did not recognize that multiplication does not always yield a larger result, and division does not always yield a smaller number.

Q5. How many decimal numbers are there between 4.3 and 4.4?
The primary purpose of this problem was to investigate whether the students understand that there are numbers between other numbers. There is a space between 4.3 and 4.4; a goal of this problem was to see whether students can relate that space with numbers. The responses are summarized in Table 6.

Table 6: Q5 Responses

Response type	Number	Percent
Correct	1	3
Incorrect	28	97

Only one student could answer this question. Her response: "I believe there is an infinite amount of decimal numbers in between 4.3 and 4.4." Twelve students (41%) answered "0.1." They subtracted 4.3 from 4.4. Nine respondents stated "one." One of their clarifications was, "one because 4.4 is only one up from 4.3." The rest of the students (8) indicated different numbers as their answers.

It is revealed from this item analysis that a large percentage of the students struggled with conceptual understanding of rational numbers. The concept that there are numbers between other numbers is an important aspect of number size understanding (Sowder, 1989).

Q6. Estimate 5575 + 3882 + 1982 + 7
This problem was designed to investigate the students' understanding of the magnitude of the relative size of numbers (quantitative judgment). Table 7 provides a summary of the findings.

Table 7: Q6 Responses

Response type	Number	Percent
Paper pencil addition	18	62
Paper pencil addition with wrong ans.	4	14
Algorithm bases	4	14
Number sense based	3	10

Eighteen students added all the numbers together and wrote "11446" without estimating. Also, another four tried to add all of the numbers together and came up with wrong answers. Four participants estimated without any explanation. The remaining three students used number sense to estimate:

6000 + 4000 + 2000 + 10 =1200, "I just rounded every number and I added them." (This student ignored 10 in her final estimation.)

5575 + 3882 + 1982 = 11439—11,000—11,400—11,440 (This student ignored 7 and wrote three estimations.)

5575 + 3882 = 10357 + 1989 = 12,346 ≈ 12,000 (she ignored 7)

The majority of students in this sample tended to heavily rely on computational techniques that had been taught in their schools. They did not even use standard algorithms to estimate the addition. Also, comparatively, a large portion of the students added all numbers together. It appears from this study that a large proportion of the students are most likely lacking good estimation skills. They also demonstrate weak quantitative judgment about the relative size of numbers, which depends on situational context. The same number can refer to a lot or a little, depending on situational contexts. Gay & Aichele (1997) explained, "Understanding a number as a quantity of a specific magnitude and being able to judge how it compares to another number is a basic to number sense" (p. 27).

7. Which is larger: 75 ÷ 0.025 or 75 ÷ 0.25?
This item attempted to examine whether or not students understood the meaning of operations by decimal numbers, and whether or not they had the ability to make a magnitude comparison between these two quotients. Their responses are summarized in Table 8.

"Number Sense" continued

Table 8: Q7 Responses

Response type	Number	Percent
Correct	16	55
Incorrect	13	45

Although slightly more than 50 percent of students responded that 75/0.025 is larger than 75/0.25, only four of them provided number-sense-based explanations, such as: "75÷ 0.025 is larger because it's a smaller decimal"; "The extra zero makes it a smaller number." Thirteen participants responded that 75 ÷ 0.25 is larger. Eight of the students were unable to explain their answers. Some of the ambiguous explanations from the students who stated that 75/0.25 is larger than 75/0.025 were: "Because 0.25 is larger than 0.025"; "Because the 0 in front of the 2 matters, it's a lower number"; "75 ÷ 0.25 because it only goes out to the hundredths place."

It appears from the students' responses that a large proportion of the sample share conceptual misunderstandings about division. Also, the absence of connection between understanding and rules was evident. They are most likely missing the concept of how many 0.025 and 0.25 they could make out of 75 (the relative size of the divisor). They may not understand that numerals are used to stand for quantities, and operations are for actions on quantities (Hicbert, 1989).

Q8. Estimate 48 percent of 500.
The main purpose of this particular item was to test the students' conceptual understanding of the percent symbol and its fractional representation. Additionally, this problem was designed to explore whether the students were capable of using benchmarks to simplify calculation. Table 9 represents the findings.

Table 9: Q8 Responses

Response type	Number	Percent
Number-sense-based reasonable estimation	13	45
Paper pencil correct multiplication	5	17
Wrong estimation or wrong multiplication	9	31
Did not answer	1	7

Thirteen participants estimated the number from 235 to 250. Among them, one notable explanation was, "Half of 500 is 250. 48% is of 100. There is 5, 10 so 2% less x 5 = 10. 240." Five students multiplied and came up with the exact answer. One student did not answer the question. The rest of the participants' estimations were not realistic. Some of them multiplied the two numbers incorrectly.

It appears from the responses that the majority of the students in this sample were lacking estimation skills, which limit their ability to assess the reasonableness of an answer. They tended to rely solely on rules and procedures to arrive at an exact answer rather than estimating. The responses from the students also uncover that a large proportion of them are most likely not familiar with using benchmarks that can be applied to simplify calculations.

9. Which is larger?
A) $\frac{1}{5} * \frac{1}{8} * \frac{1}{2} * \frac{1}{4}$

B) $\frac{1}{5} * \frac{1}{32}$

The goal of this item was to test whether the students were familiar with the flexibility of substituting different representations of numbers for a quantitative judgment. Table 10 summarizes their responses.

Table 10: Q9 Responses

Response type	Number	Percent
Correct identification (A < B)	10	35
Incorrect identification (A > B)	17	27
Incorrect identification (A = B)	2	7

Out of the 10 correct responses, only four students used number-sense-based strategies to make the correct quantitative discrimination between the two statements. They clarified that 1/2 is larger than 1/5. Two multiplied the fractions together to make the statement, and four did not write any reasoning for their answers.

In contrast, 18 students thought that 1/5 * 1/8 * 1/2 * 1/4 is larger than 1/5 * 1/32. Out of these 18, 10 stated that A is larger than B because when they multiplied the fractions together the denominator of A is larger than B. Eight students did not have any further clarifications for identifying A > B. Two students indicated the two statements are equal.

The above analysis on this item suggests that these students struggle with quantitative judgment, especially with regard to fractions. They most likely prefer to follow a routine algorithm and are incapable of using numbers in flexible ways to simplify comparison.

"Number Sense" continued

10. Is A equal to B? Why or why not?
A) 4 x 18 x 5 x 12

B) 5 x 9 x 24 x 4

This problem was also designed like the previous one to investigate the students' quantitative judgment with respect to whole numbers. Another purpose was to test whether they could use meaningful and flexible ways to identify different representations of a number for mental calculation. Table 11 presents a summary of findings.

Table 11: Q10 Responses

Response type	Number	Percent
Equal	15	52
Not equal	14	48

Out of these 15 correct responses, only two students demonstrated good number-sense- based explanations: "Yes, they are equal. Both have 4 and 5, then A has 18 and 12, B has 9, 24. Multiply one from each by 2 = same." "Yes, they are because 19*12 and 9*24 both equal 216 and 5*4 are in both." Another response was: "A = 4*320, B = 4*320, A = B. Eight students multiplied and arrived at the correct answer. Out of the 14 incorrect responses, eight multiplied inaccurately. The remaining six students gave various reasons for their conclusion. Interestingly, three students stated that the answers are not equal due to multiplying different numbers.

It appears that some students were skilled in paper-and-pencil computations, especially with whole numbers, but not skilled in their use of non-computational approaches that rely on number sense. However, their performance on this item was better than the previous item, which dealt with fractions.

The study further investigated the performance of students based on their gender, race and generation in college. The 10 number sense-based problems were graded out of 100 which revealed that male and female students did not differ significantly in their performance on number-sense test items (female students' average score =24% and male students' average score = 25%). White students performed slightly better than other races (white students' average score =27%, 15 students; other races average score = 24%, 14 students). Interestingly, second or multi-generation college students did not perform better than first generation college students (first generation average score = 29%, 17 students; and multigenerational average score = 18% 11 students; one student did not answer the question).

Conclusion
The researcher conducted this study in a department that offers two developmental mathematics courses based on students' standardized Compass test scores. The lower level of these two courses is "Basic Mathematics," which reviews arithmetic skills and their applications. The higher level of developmental mathematics course is "Elements of Algebra" which reviews basic algebra. The students in this study were enrolled in the algebra course entitled Elements of Algebra which teaches symbolic mathematics and abstract reasoning skills, and which requires a solid conceptual understanding of basic mathematics.

However, after analyzing students' work from the sample in this study, it appears that they most likely do not have a solid foundation of basic arithmetic skills, especially with regard to number sense. They struggle with the most fundamental concepts of the base ten numeration system and place value. Significantly, a large percentage of them demonstrated the following characteristics: poor understanding of rational numbers; inability to make good quantitative judgments; lack of reasoning skills; reliance solely on rule-based procedures to arrive at an answer; deficiency in estimation skills; and inability to perform mental calculations. Only 29 students participated in this study. However, based on this sample, male and female students did not differ significantly in their performance on 10 number-sense problems. Data also suggest that white students performed slightly better than other races, and second or multi-generation college students did not perform better than first generation college students in this study.

Why do most of these students lack number sense? It may be a direct consequence of curriculum, especially in secondary schools. Students understand mathematics depending on the way they were taught, which may have been mainly through procedures and habits (Faulkner, 2009; Palha et al., 2013). Written rule-driven computation limits mathematical thinking and understanding and hinders development of number sense. Dunphy (2007) found too much emphasis on assessment in public schools and almost no discussion of content. Too often students have been taught how to derive a correct answer but have not been encouraged to explain the logic behind the solution (Wu, 2011). Other contributing factors include classroom practices that do not provide or emphasize mental computation and verbal reasoning or that do not teach mathematics in the context of real life. Also, many instructional practices do not include providing appropriate feedback to students (Gersten & Chard, 1999).

"Number Sense" continued

What can developmental mathematics instructors do to promote number sense among their students? Developmental mathematics classes tend to cover too many topics in one semester, which may result in less student engagement. With this in mind, developmental mathematics educators need to revisit their mathematics curriculum. It is suggested that covering fewer topics in depth may improve students' conceptual learning outcomes. It is also suggested that instructors include more classroom practices that encourage students to develop multiple procedures to solve problems and invite students to invent these multiple procedures. Classroom environments should emphasize mental computations and verbal reasoning for these computations, and educators should invite their students to share and discuss these mental computations and solutions with other students. Instructors should be encouraged to question, facilitate, and engage rather than model solutions to their students. In addition, it is apparent from this study that there is a need for more collaboration and discussion among mathematics educators, curriculum developers, and textbook writers, test developers and researchers to find ideas and ways that may improve number sense among developmental mathematics students. There is a new trend of redesigning developmental mathematics to achieve better learning outcomes for students, and continued research on this matter is vital. Some colleges in the nation are adapting self-paced computer aided instruction (emporium models) for this redesign. Time will tell whether this emporium model improves students' quantitative ability or number sense.

Limitations

This study is based on students who took a developmental algebra course at a rural university in Pennsylvania. Its representativeness is limited. Caution should be exercised in generalizing the results. However, the data suggest that the mathematical problems in this study may be useful in assessing and monitoring developmental mathematics students' level of understanding in number sense.

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Dual Rubrics and the Process of Writing: Assessment and Best Practices in a Developmental English Course

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Abstract

This article presents strategies for using two types of essay-writing rubrics in a developmental English class of students transitioning into college-level writing. One checklist rubric is student-facing, designed to serve as a guide for students throughout the writing process and as a self-assessment tool. The other checklist rubric is instructor-facing, designed to serve as an assessment tool for instructors to provide enhanced feedback while streamlining the grading process. Additionally, this article includes some student-centered best practices for integrating the assessment process into the before, during, and after stages of the writing process.

Decades ago as a novice high school English teacher, I spent *endless* hours writing comments on my students' papers, thinking that I was providing them with helpful feedback. No matter the quality of the paper, I felt that I owed it to each student to note what was done successfully and what needed more work or could be improved upon for the next assignment. (My approach may have been a reaction to my own graded college papers, typically returned with a paucity of feedback, perhaps a holistic comment and an annotation here and there.) I enjoyed writing praise on my students' papers, but it took time to develop ways to more artfully critique writing problems so as not to discourage students.

In the years that I have been teaching writing at the college level, I have used a variety of rubrics, some of which I devised and some of which came with the course. I have certainly found that using a rubric is better than not, but most had their limitations, including being too concise or ambiguous to accurately convey clear, cogent, helpful feedback, which meant that I often supplemented those rubrics with extended comments on my students' papers.

In 2012, I read Vicki Spandel's book *Creating Writers Through 6-Trait Writing: Assessment and Instruction (5th Edition)* (2009). In her numerous publications, she provides a variety of writing guides and checklists appropriate for student writers and teachers across grades K through 12. The six traits—organization, ideas, sentence fluency, word choice, voice, and conventions—are the bases for most writing assessment, and are easily adjusted to the appropriate level of instruction. What was new to me, however, was Spandel's paradigm of corresponding writing rubrics: one that is student-facing and another that is instructor-facing. Although Spandel's target audience is neither the college student nor the college instructor, I recognized how the dual rubric approach to assessment would be advantageous for my students who are transitioning into college-level writing.

The purpose of this article has two objectives for teaching and assessing writing in developmental English at the post-secondary level. One is to present the benefits of using corresponding checklist rubrics. These rubrics can enhance the writing process for student writers transitioning into college-level writing, and these same rubrics can enhance and streamline the assessment process for instructors to provide meaningful feedback. The second objective is to present strategies for synchronizing the process of assessment with the before, during, and after stages in the process of writing.

Part I: Dual rubrics by design

Three design concepts evident in the variety of rubrics in Spandel's book *Creating Writers Through 6-Trait Writing: Assessment and Instruction (5th Ed.)* are particularly strategic to the assessment process: the use of parallel rubrics, one that is student-facing and one that is instructor-facing; the instructional, performance-based language describing each of the six writing traits; and the check-box simplicity for students to assess their own writing and for the instructor to assess according to a high-, medium-, or low-level of accomplishment. With Spandel's design in mind, I created dual rubrics for my developmental English course.

A checklist rubric for writers

What constitutes strong writing often eludes the student in transitional studies. Providing the student with a carefully crafted rubric that is student-facing can assist the student throughout the writing process and provide a number of benefits that promote learning:

- The rubric serves as a writing guide when it is provided at the time the assignment is given.
- The language in the rubric pinpoints expectations for each trait.
- The performance levels for each trait direct the student toward revision.

The self-assessment process encourages student ownership of his or her performance

"Dual Rubrics" continued

- The shared language of both student- and instructor-facing rubrics can facilitate student-instructor dialogue regarding the student's writing.

If the student is not provided with a rubric at the time an assignment is given and only learns after the paper is returned what was important for the evaluation, it seems the student has not been well served. As reported in *How Learning Works: Seven Research-Based Principles for Smart Teaching* (2010), "Research has shown that clearly specified performance criteria can help direct students' practice and ultimately their learning. For example, Andrade (2001) found that creating a rubric (a clear description of the characteristics associated with different levels of performance) and sharing it with students when an assignment is distributed leads to better outcomes—both in terms of the quality of work produced and students' knowledge of the qualities associated with good work" (Ambrose, Bridges, DiPietro, Lovett, and Norman, p. 130).

Putting the rubric into the student's hands up front, removes any mystery associated with what constitutes a successful paper and successful evaluation. Precise language in the rubric not only directly describes each component part but also, within each part, describes how the student can determine what is required for each of the three performance levels, such as whether or not his sentences are varied in structure for a high performance or whether his sentences are mostly written in the same type of structure, resulting in a lower evaluation. When students can see the differences on the checklist, they have the opportunity to take action, make revisions, and aim for a higher level of achievement in each component part. "When rubrics are given to students with the assignment description, they can help students monitor and assess their progress as they work toward clearly indicated goals" (Ambrose, Bridges, DiPietro, Lovett, and Norman, 2010, p. 232).

As implied, a student-facing rubric is written from the first-person-point-of-view. When the students assess their own writing, they are checking the boxes describing their perceived level of accomplishment for each of the traits. For example, when evaluating the introduction in a paper, a student in my development English class has these choices:

Check one of the following:

- ☐ My introduction is interesting and engages the reader in my topic.
- ☐ My introduction includes some information related to my topic, but I have not attempted a strategy to engage the reader.
- ☐ My introduction identifies my topic, but I need to include information related to my topic to engage the reader.

As students check the appropriate box, they are taking ownership of their writing and have an awareness of how their writing aligns with the expectations. If students rate themselves below the top level, they still know what is required to reach that top level, perhaps the next time. When students place the check marks in the boxes, they have taken the responsibility of assessing their writing, which entails ownership of their own performance.

Finally, the student-facing rubric prepares the student to receive the teacher's assessment. Discussions that ensue throughout the writing process are easier for both the student and the instructor who can talk the same language about targeted criteria. When the student has his or her own rubric in hand during the drafting of the essay, the student and the instructor can discuss specifics and point to that component on the rubric, such as, "My ideas are general statements on the topic without providing enough meaningful examples and specific details." The student and the instructor can have a meaningful discussion regarding what to do and strategies for how to do it. After a paper has been graded and returned, the student can compare the self-assessment with the instructor's assessment and ask targeted questions where further explanation may be required. Both the student and the teacher are able to used shared terminology, increasing the level of understanding while limiting misunderstandings (e.g., *I didn't know what we had to do,* or *I didn't know what you meant by sentence fluency*).

A checklist rubric for writing assessment: The instructor-facing rubric

Assessing the writing of transitioning students has its challenges in that in order to be helpful, we instructors must first aim to do no harm. Giving constructive feedback that avoids negativity is essential. The parallel alignment of dual rubrics assists in that goal. So what are the advantages of an instructor-facing checklist rubric for assessing student writing?

- Establishment of clear, targeted requirements
- Promotion of clear communication of the writing objectives with the student throughout the writing process
- Provision of the means for clear, neutral feedback
- Improvement in the post-assessment dialogue between the student and the instructor

"Dual Rubrics" continued

- Reduction in the time spent evaluating and grading student writing

The instructor-facing assessment rubric aligns with the components and the criteria in the writer's rubric and additionally includes a third column for point distribution for each of the six traits. Each line on the student-facing rubric has a comparable line on the instructor-facing rubric. However, the significant difference between the two checklists is the more objective language on the instructor-facing rubric, which assesses the student's writing, not the writer. So for example, these are the comparable checklist choices on the instructor-facing rubric for evaluating the essay introduction:

- ☐ The introduction is interesting and engages the reader in the topic.
- ☐ The introduction includes some information related to the topic but needs a strategy to engage the reader.
- ☐ The introduction identifies the topic but needs to include information to engage the reader in the topic.

The emphasis of the feedback for this section is on the introduction and the phrasing is neutral. Whereas this section in the student-facing rubric uses the personal pronoun *my*, as in *my introduction* and *my point*, the instructor's assessment rubric uses objective wording, such as *the i*ntroduction and *the* point.
The description of what constitutes high achievement is fairly straight forward; however, phrasing less than top-notch performance requires more finesse. It is far more effective to check a box with a neutral tone to state that "the introduction identifies the topic but needs to include information and to engage the reader in the topic" rather than using the second-person: "*Your* introduction does not do enough to introduce *your* topic."

Further, because the phrasing for medium- or low-quality provides a pathway to improvement, the rubric does serve as a learning tool. When a student reads that "some of *the* sentences are clearly worded while others are not" and "more specific word choices are needed to replace general or vague words," the message is that some elements can be fixed/revised/worked on for next time and that it is within the student's power to do so. Because students checked the appropriate boxes on the rubric when they completed their own assessment, they may have a better understanding and acceptance that the grading process is fair. They can note that the instructor assessed the same qualities in the essay.

During the 2013 spring session, I used the check-box rubrics for both essay assignments in the course. At the end of the session, I surveyed the students to arrive at qualitative results for the effectiveness of using these rubrics as teaching tools. I was pleased, and not surprised, by the answers and feedback. The survey follows, along with the results.

Check-box rubrics for writers: A survey for the May 2013 session
Please place an ***X*** next to either "Yes" or "No" for each of the following questions regarding the rubrics that were used for Essay 1 and Essay 2 this session.

1. Were the descriptions next to each check box on the Rubric for Writers clearly worded so that you understood what you were checking for each category, such as for organization, supporting ideas, sentence fluency, etc.

   _____ Yes _____ No

   Comments:

2. Did having the assessment categories and descriptions listed as checkboxes on the Rubric for Writers provide a guide for you as you drafted and/or revised your essay to fulfill the assignment?

   _____ Yes _____ No

   Comments:

3. Did you already know what to expect regarding how your instructor would evaluate your essay because you had completed the check-box rubric?
   _____ Yes _____ No
   Comments:
4. Do you think that having a check-box rubric to guide you before you complete your assignments would be helpful to you in fulfilling writing assignments in the future?

   _____ Yes _____ No

   Comments:

Results
Three students (out of four) completed the course and were present on the last day of class to take the survey:
Question 1: Yes = 3; No = 0

Comments: None

Question 2: Yes = 3; No = 0

Comments: "I enjoyed going over the check box to see if you agree with what I marked."

Question 3: Yes = 3; No = 0

Comment: "This helped me to organize my papers."

Question 4: Yes = 3; No = 0

"I plan on looking for these rubrics check boxes when I take English 112 in the fall session (hopefully with you)!"

"Dual Rubrics" continued

The most current iterations of both rubrics appear at the end of this article in the appendices.

Part II: Best practices within the process of assessing

Students are well served when assessing their writing is integrated throughout their writing process. For transitional students in developmental English, such an approach positions the instructor to scaffold the students along the way. I think of the process of assessing as a series of contact points with students as they plan, compose, and revise their writing. The use of rubrics supports that approach.
The following section provides tips and strategies.

Assessing in the prewriting stage

While students are in the prewriting stage, this is my typical approach:

- Tell students what I expect—in writing. Crafting clear assignments is the first step, with the inclusion of requirements and expectations.
- Provide students with a Checklist Rubric for Writers so they know from the start precisely how their writing will be evaluated. Ambrose, Bridges, DiPietro, Lovett, and Norman (2010) underscore the importance of the rubric as a guide: "Rubrics are a way of explicitly representing performance expectations and thus can direct students' behaviors toward your intended goals" (p. 87).
- Provide an activity involving model essays for discussion of the traits.
- Provide students with an Idea Map (an outline template) to assist them in organizing their major points and key ideas for support.

During the drafting stage

While students are drafting their papers, this is my typical approach:

- Conduct individual mini-conferences with students during designated workshop time to discuss their plans, initial drafts, and progress. Checking early, such as after the student has composed a couple of paragraphs, can prevent later headaches. Allowing as little as five minutes per student can be productive. For short essays, these conversations allow me to make verbal suggestions and eliminate the need for "grading" the draft, which is by its nature, a work in progress anyway.
- Require students to complete the Checklist Rubric for Writers, checking off the appropriate boxes as they proceed. The Rubric for Writers is stapled to the final draft at the time of submission.

After submission

When grading student papers, this is my typical approach:

- Use a Checklist Rubric for Writing Assessment, which parallels the Checklist Rubric for Writers.
- Include at least one or two additional salient, positive remarks; targeted praise is motivating. For comments on problem spots, posing questions can circumvent criticism, such as, *what is another example that would help to support your point?*
- Limit the editing. I often focus on the first paragraphs or page to note a few areas where the student needs to work on matters of accuracy, especially those that we have already worked on to date. Often, I just underline the problem, such as a matter of diction or the point where a sentence is run-on or place brackets around a sentence fragment. The student can then contemplate the notations for discussion in our post-paper mini-conference. At the point where I discontinue editing marks, I write the phrase *my editing ends here* in the margin. Too much editing can overwhelm the student, but by alerting the student where I have stopped editing avoids giving an impression that the successive paragraphs are flawless.
- Avoid using red ink (though the word rubric actually derives from the Latin word *ruber*, meaning red) (Taylor, 2009, para. 2). I use green, blue, or occasionally, purple. Those colors seem to convey that I am making comments, not spilling blood. I even mention to students that research shows that using red ink to grade papers can have damaging psychological results. Some student expressions in response seem to relate to that. According to the *Journal of College Science Teaching,* a research study conducted by psychology professor Andrew Elliot (2007) at the University of Rochester found that there is a specific association between red and mistakes and failures of people. When a teacher uses red ink to mark a student's paper it can have a negative impact on behavior (p. 8).
- Use a checklist rubric with the writing traits and levels of performance clearly stated. The rubric eliminates or lessens the need for lengthy comments, thus streamlining the grading process while providing meaningful feedback.
- Hold post-grading, mini-conferences with students to discuss their graded papers in a conversation geared to answer questions and to frequently serve as a positive segue way into the next paper.

This before-during-and-after process establishes a familiar rhythm in the classroom. Working with students throughout the process fosters a sense of collaboration.

"Dual Rubrics" continued

Conclusion

Confusing feedback or the lack of feedback can be disappointing for the student and a lost teaching opportunity for the instructor. Consistent and purposeful interaction with our student writers to guide their efforts from the practice stages to printed/published performance paves the path toward reaching the desired outcomes. During a recent session of my developmental English course, I asked my four students at various checkpoints along the writing process how they were using the checklist. The following represents the essence of their responses:

Student 1:

- *I made changes to my introduction to make the reader interested.*
- *I realized that my conclusion did not do enough to give something to think about. I will work on that next time.*
- *I have had rubrics before, but I liked having one for me so I can say what I think about my paper.*

Student 2:

- *It (the checklist) made me work harder.*
- *I went over my essay three times as I thought about the checklist.*
- *I graded myself harder than you (referring to me, the instructor) did.*
- *I would like a blank copy to keep.*

Student 3:

- *I revised a lot, especially to give more specific examples.*

Student 4:

- *I changed two sentences in my conclusion to improve my voice.*

"If you present clear and expansive ideas, I will understand; if you organize information effectively, I will follow; if you write with voice, I will hear you" (Spandel, 2009, p. 29). This inspirational message ignited my desire to create student-centered assessment instruments for the novice writers in my classes. Checklist assessment enhances the process of guiding students toward expanding, organizing, and voicing their ideas. When their self-assessment becomes metacognitive, then novice writers are better prepared to transition into college-level writing.

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Appendix: A Checklist Rubric for Writers

Use this checklist as a guide as you draft, revise, and proofread your paper. Then, when you turn in the final copy, include this rubric with a check mark in the appropriate box for each section to show your assessment of your own writing.

Organization

Check one of the following:

- ☐ My introduction is interesting and engages the reader in my topic.
- ☐ My introduction includes information related to my topic, but I have not attempted a strategy to engage the reader.
- ☐ My introduction identifies my topic, but I need to include information related to my topic and to engage the reader.

- ☐ My thesis is stated in the introduction. It clearly identifies the point that I want to make about my topic.
- ☐ My topic is stated in the introduction, but I do not have a thesis statement that clearly identifies my point for the paper.
- ☐ My main point is missing in my introduction.

- ☐ Each paragraph in the body of my essay has a focused topic sentence/point that supports my thesis statement.
- ☐ Some of the paragraphs in the body of my essay still need a focused topic sentence/point that supports my thesis statement.
- ☐ The paragraphs in the body of my essay do not have topic sentences/points that support my thesis statement.

- ☐ My conclusion expresses the significance of my ideas and leaves an impression on the reader.
- ☐ My conclusion repeats some of the same ideas explained in the body of my essay without giving their significance.
- ☐ My conclusion does not give a sense of closure.

Supporting ideas

- ☐ My ideas on the topic are thoughtful and support my points with specific examples and details.
- ☐ Some of my ideas are thoughtful, though some support is too general or repetitious.

"Dual Rubrics" continued

- ☐ My ideas are general statements on the topic without providing enough thoughtful examples and specific details.

Sentence fluency & word choice

- ☐ My sentences are varied, clearly worded, and include many precise nouns and verbs.
- ☐ Some of my sentences are clearly worded while others are not. Some of my nouns and verbs are precise.
- ☐ Many or most of my sentences have the same sentence structure and many of my word choices are general or vague terms.

- ☐ I have included transitions at the beginning of paragraphs and within paragraphs to effectively connect my ideas.
- ☐ I have included some effective transitions, but additional transitions would improve the connections between my ideas.
- ☐ I have not included enough or effective transitions to smoothly connect my ideas for the reader to follow.

Voice

- ☐ My writing sounds like me as the person engaged with my topic. I have a presence on the page beginning with the introduction, throughout the body of the paper, and in the conclusion.
- ☐ My writing sounds like me in some parts, but in some parts it sounds unnatural or awkward.
- ☐ My paper pertains to the assignment, but the writing does not show my engagement with the topic.

Standard English conventions

- ☐ My writing has a high level of accuracy, including standard capitalization, spelling, punctuation, verb tense, and pronoun usage.
- ☐ My writing is mostly accurate in the areas of standard capitalization, spelling, punctuation, verb tense, and pronoun usage.
- ☐ My writing still needs more of my attention to accuracy in order to effectively communicate my ideas to the reader.

Appendix B: A checklist rubric for essay assessment (100 pts.)

Organization (20 pts.)

- ☐ The introduction is interesting and engages the reader in the topic.
- ☐ The introduction includes some information related to the topic but it needs a strategy to engage the reader.
- ☐ The introduction identifies the topic but needs to include information related to the topic and to engage the reader.

- ☐ The thesis is stated in the introduction. It clearly identifies the writer's point regarding the topic.
- ☐ The topic is stated in the introduction, but the point regarding the topic still needs to be stated clearly.
- ☐ The main point is missing in the introduction.

- ☐ Each paragraph in the body of the essay has a focused topic sentence/point that supports the thesis statement.
- ☐ Some paragraphs in the body of the essay still need a focused topic sentence/point that supports the thesis.
- ☐ The paragraphs in the body of the essay need topic sentences/points that support the thesis statement.

- ☐ The conclusion expresses the significance of the writer's ideas and leaves an impression on the reader.
- ☐ The conclusion repeats some of the same ideas explained in the body of the essay without stating their significance.
- ☐ The conclusion needs to give a sense of closure.

Supporting ideas (40 pts.)

- ☐ The ideas on the topic are thoughtful and support the points with specific examples and details.
- ☐ Some of the ideas are thoughtful, though some support may be too general or repetitious.
- ☐ The ideas are general statements on the topic without providing specific examples and details.

Sentence fluency & word choice (15 pts.)

- ☐ The sentences are varied, clearly worded, and include many precise nouns and verbs.
- ☐ Some of the sentences are clearly worded while others are not. Some of the nouns and verbs are precise.
- ☐ Many or most of the sentences have the same sentence structure and more precise nouns and verbs are needed to replace general or vague terms.

"Dual Rubrics" continued

- ☐ Transitions are included at the beginning of paragraphs and within paragraphs effectively connecting the ideas.
- ☐ Some transitions effectively connect ideas, though additional transitions would improve the flow between ideas.
- ☐ More effective transitions are needed to smoothly connect the ideas for the reader.

Voice (10 pts.)

- ☐ The writing sounds like the writer who is engaged with the topic and has a presence on the page beginning with the introduction, throughout the body of the paper, and in the conclusion.
- ☐ The writing sounds like the writer in some parts, but in other parts it sounds unnatural or awkward.
- ☐ The paper pertains to the assignment, but the writing needs more of the genuine voice of the writer to show engagement with the topic.

Standard English conventions (15 pts.)

- ☐ The writing has a high level of accuracy, including standard capitalization, spelling, punctuation, verb tense, and pronoun usage.
- ☐ The writing is mostly accurate in the areas of standard capitalization, spelling, punctuation, verb tense, and pronoun usage.
- ☐ The writing still needs a higher level of accuracy to effectively communicate the ideas to the reader. Recommendation: work with a campus or online writing tutor.

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